Kalman filter: linear GAUSSian likelihood/dynamics, $\mathbf{x}_k = (\mathbf{r}_k^\top, \dot{\mathbf{r}}_k^\top, \ddot{\mathbf{r}}_k^\top)^\top$, $\mathcal{Z}^k = \{\mathbf{z}_k, \mathcal{Z}^{k-1}\}$

initiation: $p(\mathbf{x}_0) = \mathcal{N}(\mathbf{x}_0;\, \mathbf{x}_{0|0},\, \mathbf{P}_{0|0})$, initial ignorance: $\mathbf{P}_{0|0}$ 'large'

prediction:

$$\mathcal{N}(\mathbf{x}_{k-1};\, \mathbf{x}_{k-1|k-1},\, \mathbf{P}_{k-1|k-1}) \xrightarrow[\mathbf{F}_{k|k-1},\, \mathbf{D}_{k|k-1}]{\text{dynamics model}} \mathcal{N}(\mathbf{x}_k;\, \mathbf{x}_{k|k-1},\, \mathbf{P}_{k|k-1})$$

$$\mathbf{x}_{k|k-1} = \mathbf{F}_{k|k-1}\mathbf{x}_{k-1|k-1}$$
$$\mathbf{P}_{k|k-1} = \mathbf{F}_{k|k-1}\mathbf{P}_{k-1|k-1}\mathbf{F}_{k|k-1}{}^\top + \mathbf{D}_{k|k-1}$$

filtering:

$$\mathcal{N}(\mathbf{x}_k;\, \mathbf{x}_{k|k-1},\, \mathbf{P}_{k|k-1}) \xrightarrow[\text{sensor model: } \mathbf{H}_k,\, \mathbf{R}_k]{\text{current measurement } \mathbf{z}_k} \mathcal{N}(\mathbf{x}_k;\, \mathbf{x}_{k|k},\, \mathbf{P}_{k|k})$$

$$\mathbf{x}_{k|k} = \mathbf{x}_{k|k-1} + \mathbf{W}_{k|k-1}\boldsymbol{\nu}_{k|k-1}, \qquad \boldsymbol{\nu}_{k|k-1} = \mathbf{z}_k - \mathbf{H}_k\mathbf{x}_{k|k-1}$$
$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{W}_{k|k-1}\mathbf{S}_{k|k-1}\mathbf{W}_{k|k-1}{}^\top, \qquad \mathbf{S}_{k|k-1} = \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k{}^\top + \mathbf{R}_k$$
$$\mathbf{W}_{k|k-1} = \mathbf{P}_{k|k-1}\mathbf{H}_k{}^\top\mathbf{S}_{k|k-1}{}^{-1}$$

'KALMAN gain matrix'

retrodiction:

$$\mathcal{N}(\mathbf{x}_l;\, \mathbf{x}_{l|k},\, \mathbf{P}_{l|k}) \xleftarrow[\text{dynamics model}]{\text{filtering, prediction}} \mathcal{N}(\mathbf{x}_{l+1};\, \mathbf{x}_{l+1|k},\, \mathbf{P}_{l+1|k})$$

$$\mathbf{x}_{l|k} = \mathbf{x}_{l|l} + \mathbf{W}_{l|l+1}(\mathbf{x}_{l+1|k} - \mathbf{x}_{l+1|l}), \qquad \mathbf{W}_{l|l+1} = \mathbf{P}_{l|l}\mathbf{F}_{l+1|l}^\top\mathbf{P}_{l+1|l}^{-1}$$
$$\mathbf{P}_{l|k} = \mathbf{P}_{l|l} + \mathbf{W}_{l|l+1}(\mathbf{P}_{l+1|k} - \mathbf{P}_{l+1|l})\mathbf{W}_{l|l+1}{}^\top$$

Continuous Time Retrodiction for $t_l < t_{l+\theta} < t_{l+1}$ with $0 < \theta < 1$

Interpolate between $p(\mathbf{x}_l|\mathcal{Z}^k)$ and $p(\mathbf{x}_{l+1}|\mathcal{Z}^k)$ based on the evolution model:

$$p(\mathbf{x}_{l+\theta}|\mathcal{Z}^k) = \int d\mathbf{x}_{l+1}\, p(\mathbf{x}_{l+\theta}|\mathbf{x}_{l+1}, \mathcal{Z}^k)\, p(\mathbf{x}_{l+1}|\mathcal{Z}^k)$$

where: $$p(\mathbf{x}_{l+\theta}|\mathbf{x}_{l+1}, \mathcal{Z}^k) = \frac{p(\mathbf{x}_{l+1}|\mathbf{x}_{l+\theta})\, p(\mathbf{x}_{l+\theta}|\mathcal{Z}^l)}{\int d\mathbf{x}_{l+\theta}\, p(\mathbf{x}_{l+1}|\mathbf{x}_{l+\theta})\, p(\mathbf{x}_{l+\theta}|\mathcal{Z}^l)}$$

Looks like a Kalman update!

with: $$\begin{aligned} p(\mathbf{x}_{l+1}|\mathbf{x}_{l+\theta}) &= \mathcal{N}\big(\mathbf{x}_{l+1};\, \mathbf{F}_{l+1|l+\theta}\mathbf{x}_{l+\theta},\, \mathbf{D}_{l+1|l+\theta}\big) \\ p(\mathbf{x}_{l+\theta}|\mathcal{Z}^l) &= \int d\mathbf{x}_l\, p(\mathbf{x}_{l+\theta}|\mathbf{x}_l)\, p(\mathbf{x}_l|\mathcal{Z}^l) \\ p(\mathbf{x}_{l+1}|\mathcal{Z}^l) &= \mathcal{N}\big(\mathbf{x}_{l+1};\, \mathbf{x}_{l+1|l},\, \mathbf{P}_{l+1|l}\big) \end{aligned}$$

Looks like Kalman prediction!

Fraunhofer FKIE

One possible sensor data fusion strategy:

Create a singe effective measurement z_k with measurement error R_k

by preprocessing of individual sensor measurements $\mathrm{z}_k^i, \mathrm{R}_k^i$, $i = 1, \ldots, S_k$!

$$\mathrm{z}_k = \mathrm{R}_k \sum_{s=1}^{S_k} \left(\mathrm{R}_k^s\right)^{-1} \mathrm{z}_k^s$$

weighted arithmetic mean

$$\mathrm{R}_k^{-1} = \sum_{s=1}^{S_k} \left(\mathrm{R}_k^s\right)^{-1}$$

harmonic mean

A typical structure for fusion equations!

Fraunhofer FKIE

Sensor data: range, azimuth, range-rate

Coordinates: Sensor data $\to$ *polar*, object evolution $\to$ *Cartesian*

Dynamics system: $\mathbf{x}^d = (x, y, \dot{x}, \dot{y})$

Sensor system: $\mathbf{x}^s = (r, \varphi, \dot{r}, \dot{\varphi})$

$$p\left(\mathbf{x}^d_{k-1} \middle| \mathcal{Z}^{k-1}\right) \xrightarrow[\text{Dynamics}]{p\left(\mathbf{x}^d_k \middle| \mathbf{x}^d_{k-1}\right)} p\left(\mathbf{x}^d_k \middle| \mathcal{Z}^{k-1}\right) \qquad p\left(\mathbf{x}^d_k \middle| \mathcal{Z}^{k}\right)$$

$$\uparrow \mathbf{t}_{d \leftarrow s} \qquad \downarrow \mathbf{t}_{s \leftarrow d} \qquad \uparrow \mathbf{t}_{d \leftarrow s}$$

$$p\left(\mathbf{x}^s_{k-1} \middle| \mathcal{Z}^{k-1}\right) \qquad p\left(\mathbf{x}^s_k \middle| \mathcal{Z}^{k-1}\right) \xrightarrow[\text{Sensor}]{p\left(\mathbf{z}_k \middle| \mathbf{x}^s_k\right)} p\left(\mathbf{x}^s_k \middle| \mathcal{Z}^{k}\right)$$

scan $k-1$ | scan k | scan k

***non-linear* coordinate transformations:**

$$\mathbf{t}_{d \leftarrow s}[\mathbf{x}^s] = \begin{pmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} r \cos\varphi \\ r \sin\varphi \\ \dot{r}\cos\varphi - r\dot{\varphi}\sin\varphi \\ \dot{r}\sin\varphi + r\dot{\varphi}\cos\varphi \end{pmatrix} \qquad \mathbf{t}_{s \leftarrow d}[\mathbf{x}^d] = \begin{pmatrix} r \\ \varphi \\ \dot{r} \\ \dot{\varphi} \end{pmatrix} = \begin{pmatrix} \sqrt{x^2+y^2} \\ \arctan y/x \\ (x\dot{x}+y\dot{y})/\sqrt{x^2+y^2} \\ (x\dot{y}-y\dot{x})/(x^2+y^2) \end{pmatrix}$$

Extended *Kalman* filter: be wise - linearize!

non-lineare transformations: Taylor expansion up to 1st order

around $\mathbf{x}^s_{k|k}$ (filtering): $\mathbf{t}_{d\leftarrow s}[\mathbf{x}^s_k] \approx \mathbf{t}_{d\leftarrow s}[\mathbf{x}^s_{k|k}] + \mathbf{T}_{d\leftarrow s}[\mathbf{x}^s_{k|k}]\,(\mathbf{x}^s_k - \mathbf{x}^s_{k|k})$

mit: $\mathbf{T}_{d\leftarrow s}[\mathbf{x}^s_{k|k}] = \partial \mathbf{t}_{d\leftarrow s}[\mathbf{x}^s_{k|k}] / \partial \mathbf{x}^s_{k|k}$ (Jacobian)

around $\mathbf{x}^d_{k|k-1}$ (Prediction): $\mathbf{t}_{s\leftarrow d}[\mathbf{x}^d_k] \approx \mathbf{t}_{s\leftarrow d}[\mathbf{x}^d_{k|k-1}] + \mathbf{T}_{d\leftarrow s}[\mathbf{x}^d_{k|k-1}]\,(\mathbf{x}^d_k - \mathbf{x}^d_{k|k-1})$

with: $\mathbf{T}_{s\leftarrow d} = \partial \mathbf{t}_{d\leftarrow s}[\mathbf{x}^d_{k|k-1}] / \partial \mathbf{x}^d_{k|k-1}$

affine transformation of Gaussian random variables:

$$\mathcal{N}\big(x;\, \mathbf{x},\, \mathbf{X}\big) \xrightarrow{y=\mathbf{a}+\mathbf{A}x} \mathcal{N}\big(y;\, \mathbf{a}+\mathbf{A}\mathbf{x},\, \mathbf{A}\mathbf{X}\mathbf{A}^\top\big)$$

A side Result: *Expected* Measurements

innovation statistics, expectation gates, gating

$$p(\mathbf{z}_k | \mathcal{Z}^{k-1}) = \int d\mathbf{x}_k \, p(\mathbf{z}_k, \mathbf{x}_k | \mathcal{Z}^{k-1}) = \int d\mathbf{x}_k \, p(\mathbf{z}_k | \mathbf{x}_k) \, p(\mathbf{x}_k | \mathcal{Z}^{k-1})$$

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$$= \int d\mathbf{x}_k\, \underbrace{\mathcal{N}\big(\mathbf{z}_k;\, \mathbf{H}_k\mathbf{x}_k,\, \mathbf{R}_k\big)}_{\text{likelihood: sensor model}}\, \underbrace{\mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{k|k-1},\, \mathbf{P}_{k|k-1}\big)}_{\text{prediction at time } t_k}$$

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$$p(\mathbf{z}_k | \mathcal{Z}^{k-1}) = \int d\mathbf{x}_k \, p(\mathbf{z}_k, \mathbf{x}_k | \mathcal{Z}^{k-1}) = \int d\mathbf{x}_k \, p(\mathbf{z}_k | \mathbf{x}_k) \, p(\mathbf{x}_k | \mathcal{Z}^{k-1})$$

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$$= \mathcal{N}\big(\mathbf{z}_k;\, \mathbf{H}_k\mathbf{x}_{k|k-1},\, \mathbf{S}_{k|k-1}\big) \qquad \text{(product formula)}$$

innovation: $\boldsymbol{\nu}_{k|k-1} = \mathbf{z}_k - \mathbf{H}_k\mathbf{x}_{k|k-1}$,

innovation covariance: $\mathbf{S}_{k|k-1} = \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^\top + \mathbf{R}_k$

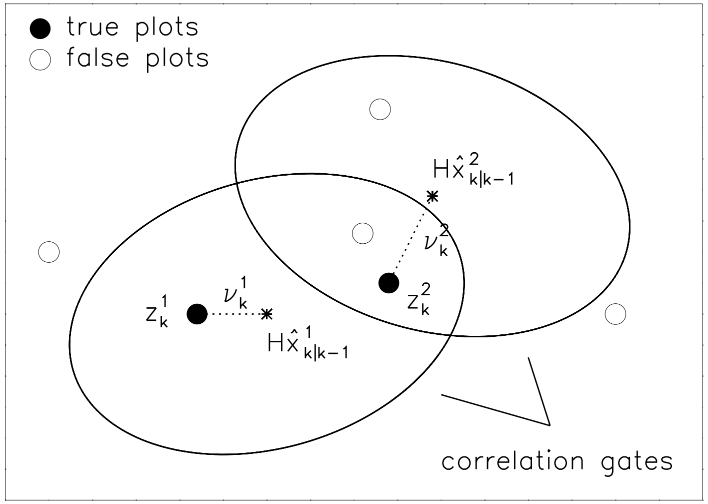
expectation gate: $\boldsymbol{\nu}_{k|k-1}^\top \mathbf{S}_{k|k-1}^{-1} \boldsymbol{\nu}_{k|k-1} \leq \lambda(P_c)$

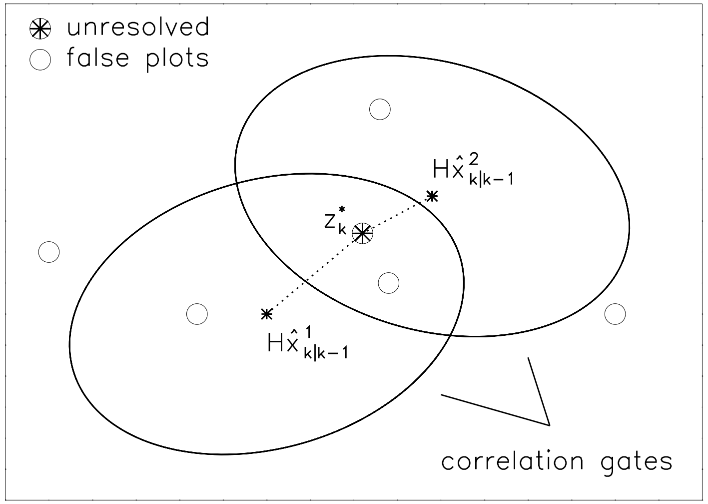
MAHALANOBIS ellipsoid containing $\mathbf{z}_k$ with certain probability P_c

Choose $\lambda(P_c)$ (“gating parameter”) properly!

Can be looked up in a χ^2-table - discussed later!

Sensor data of uncertain origin



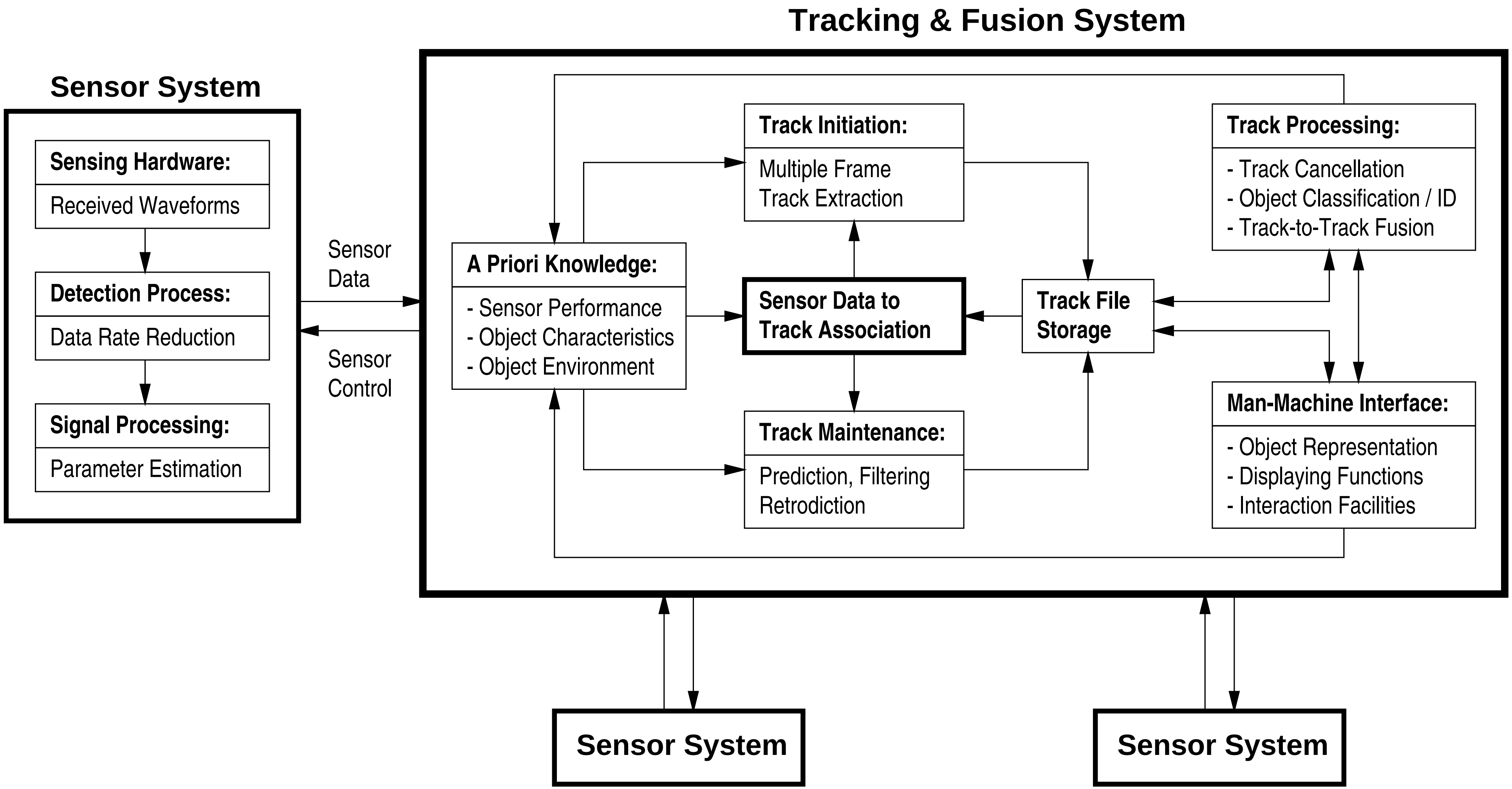


- prediction: $\mathbf{x}_{k|k-1}$, $\mathbf{P}_{k|k-1}$ (dynamics)
- innovation: $\boxed{\boldsymbol{\nu}_k = \mathbf{z}_k - \mathbf{H}\mathbf{x}_{k|k-1}}$, white
- Mahalanobis norm: $\boxed{||\boldsymbol{\nu}_k||^2 = \boldsymbol{\nu}_k^\top \mathbf{S}_k^{-1} \boldsymbol{\nu}_k}$
- expected plot: $\mathbf{z}_k \sim N(\mathbf{H}\mathbf{x}_{k|k-1}, \mathbf{S}_k)$
- $\boldsymbol{\nu}_k \sim N(0, \mathbf{S}_k)$, $\boxed{\mathbf{S}_k = \mathbf{H}\mathbf{P}_{k|k-1}\mathbf{H}^\top + \mathbf{R}}$
- gating: $||\boldsymbol{\nu}_k|| < \lambda$, $P_c(\lambda)$ correlation prob.

missing/false plots, measurement errors, scan rate, agile targets: large gates

Fraunhofer FKIE

A Generic Tracking and Sensor Data Fusion System



Fraunhofer FKIE

Description of the Detection Process

Detector: receives signals and decides on object existence

Processor: processes detected signals and produces measurements

'D': detector detects an object

D: object actually existent

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error of 1. kind: $P_{\mathrm{I}} = P(\neg\text{‘}D\text{’}|D)$

error of 2. kind: $P_{\mathrm{II}} = P(\text{‘}D\text{’}|\neg D)$

measure of detection performance: $P_D = P(\text{‘}D\text{’}|D)$

detector properties characterized by two parameters:

– detection probability $P_D = 1 - P_{\mathrm{I}}$
– false alarm probability $P_F = P_{\mathrm{II}}$

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Detector: receives signals and decides on object existence

Processor: processes detected signals and produces measurements

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detector properties characterized by two parameters:

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example (Swerling I model): $P_D = P_D(P_F, \text{SNR}) = P_F^{1/(1+\text{SNR})}$

detector design: Maximize detection probability P_D for a given, predefined false alarm probability P_F!

ambiguous sensor data ($P_D < 1$, $\rho_F > 0$)

$n_k + 1$ possible interpretations of the sensor data $Z_k = \{\mathbf{z}_k^j\}_{j=1}^{n_k}$!

- E_0: the object was not detected; n_k false data in the Field of View (FoV)
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Fraunhofer FKIE

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$$= p(Z_k, n_k|\neg D, \mathbf{x}_k) \underbrace{P(\neg D|\mathbf{x}_k)}_{=1-P_D} + p(Z_k, n_k|D, \mathbf{x}_k) \underbrace{P(D|\mathbf{x}_k)}_{=P_D}$$

sensor parameter: detection probability P_D

Fraunhofer FKIE

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$$= p(Z_k, n_k|\neg D, \mathbf{x}_k)\, P(\neg D|\mathbf{x}_k) + p(Z_k, n_k|D, \mathbf{x}_k)\, p(D|\mathbf{x}_k)$$

$$= \underbrace{p(Z_k|n_k, \neg D, \mathbf{x}_k)}_{=|\text{FoV}|^{-n_k}} \underbrace{p(n_k|\neg D, \mathbf{x}_k)}_{=p_F(n_k)} (1 - P_D) + P_D \sum_{j=1}^{n_k} p(Z_k, n_k, j|D, \mathbf{x}_k)$$

false measurements: **Poisson distributed in #, uniformly distributed in the** FoV

Modeling of False Measurements (FM)

- **Probability of having n FM:** $p_F(n)$

  – mean number of FM in the 'Field of View' (FoV) of a sensor:

    $\bar{n} = \rho_F|\mathsf{FoV}|$, false measurement density ρ_F (perhaps not constant)

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    expectation: $\mathbb{E}[n] = \bar{n}$, variance: $\mathbb{V}[n] = \bar{n}$

Fraunhofer FKIE

normalization: $\sum_{n=0}^{\infty} p_F(n) = \mathrm{e}^{-\bar{n}} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{n!} = \mathrm{e}^{-\bar{n}}\, \mathrm{e}^{\bar{n}} = 1$

expectation: $\mathbb{E}[n] = \mathrm{e}^{-\bar{n}} \sum_{n=0}^{\infty} n \frac{\bar{n}^n}{n!} = \mathrm{e}^{-\bar{n}} \sum_{n=1}^{\infty} n \frac{\bar{n}^n}{n!} = \bar{n}\mathrm{e}^{-\bar{n}} \sum_{n=1}^{\infty} \frac{\bar{n}^{n-1}}{(n-1)!} = \bar{n}$

variance: $\mathbb{V}[n] = \mathbb{E}[(n-\bar{n})^2] = \mathbb{E}[n^2] - \bar{n}^2 = \ldots$ exercise! $\ldots = \bar{n}$

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- **Distribution of FM in the Field of View:** $p(\mathbf{z}_1^f, \ldots, \mathbf{z}_n^f|\textbf{FoV})$
  - FM mutually independent: $p(\mathbf{z}_1^f, \ldots, \mathbf{z}_n^f|\text{FoV}) = \prod_{i=1}^{n} p(\mathbf{z}_i^f|\text{FoV})$

Fraunhofer FKIE

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  - uniformly distributed in the FoV: $p(\mathbf{z}_i^f|\mathsf{FoV}) = |\mathsf{FoV}|^{-1}$ (often!)

ambiguous sensor data ($P_D < 1$, $\rho_F > 0$)

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$$= p(Z_k, n_k|\neg D, \mathbf{x}_k)\, P(\neg D|\mathbf{x}_k) + p(Z_k, n_k|D, \mathbf{x}_k)\, p(D|\mathbf{x}_k)$$

$$= p(Z_k|n_k, \neg D, \mathbf{x}_k)\, p(n_k|\neg D, \mathbf{x}_k)\, (1 - P_D) + P_D \sum_{j=1}^{n_k} p(Z_k, n_k, j|D, \mathbf{x}_k)$$

$$= |\text{FoV}|^{-n_k}\, p_F(n_k)\, (1 - P_D) + P_D \sum_{j=1}^{n_k} \underbrace{p(Z_k|n_k, j, D, \mathbf{x}_k)}_{|\text{FoV}|^{-(n_k-1)} N(\mathbf{z}_k^j; \mathbf{H}\mathbf{x}_k, \mathbf{R})} \underbrace{p(j|n_k, D)}_{=1/n_k} \underbrace{p(n_k|D)}_{=p_F(n_k-1)}$$

Insert Poisson distribution: $p_F(n_k) = \frac{(\rho_F|\text{FoV}|)^{-n_k}}{n_k!}\, \mathrm{e}^{-\rho_F|\text{FoV}|}$

Fraunhofer FKIE

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$$= |\text{FoV}|^{-n_k}\, p_F(n_k)\, (1 - P_D) + P_D \sum_{j=1}^{n_k} p(Z_k|n_k, j, D, \mathbf{x}_k)\, p(j|n_k, D)\, p(n_k|D)$$

$$= \frac{e^{-\rho_F|\text{FoV}|}}{n_k!}\, \rho_F^{n_k-1}\Big((1 - P_D)\rho_F + P_D \sum_{j=1}^{n_k} \mathcal{N}\big(\mathbf{z}_k^j;\, \mathbf{H}\mathbf{x}_k,\, \mathbf{R}\big)\Big)$$

Fraunhofer FKIE

Likelihood Functions

The likelihood function answers the question:

What does the sensor tell about the state $\mathbf{x}$ of the object?

(input: sensor data, sensor model)

- **ideal conditions, one object:** $P_D = 1$**,** $\rho_F = 0$

  at each time one measurement: $$p(\mathbf{z}_k|\mathbf{x}_k) = \mathcal{N}(\mathbf{z}_k; \mathbf{H}\mathbf{x}_k, \mathbf{R})$$

- **real conditions, one object:** $P_D < 1$**,** $\rho_F > 0$

  at each time n_k measurements $Z_k = \{\mathbf{z}_k^1, \dots, \mathbf{z}_k^{n_k}\}$!

$$p(Z_k, n_k|\mathbf{x}_k) \propto (1 - P_D)\rho_F + P_D \sum_{j=1}^{n_k} \mathcal{N}\big(\mathbf{z}_k^j;\, \mathbf{H}\mathbf{x}_k,\, \mathbf{R}\big)$$