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- **Problem:** imperfect sensor information: inaccurate, incomplete, and eventually ambiguous. Moreover, the targets' temporal evolution is usually not well-known.
- **Approach:** Interpret measurements and state vectors as random variables (RVs). Describe by probability density functions (pdf) what is known about them.
- **Solution:** Derive iteration formulae for calculating the pdfs! Develop a mechanism for initiation! By doing so, exploit all background information available! Derive state estimates from the pdfs along with appropriate quality measures!

Bayesian Multiple Sensor Tracking: Basic Idea

Iterative updating of conditional probability densities!

kinematic target state x_k at time t_k, **accumulated multiple sensor data** $\mathcal{Z}^k$

a priori knowledge: target dynamics models, sensor model, other context

- **prediction:** $p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1}) \xrightarrow[\text{context}]{\text{dynamics model}} p(\mathbf{x}_k|\mathcal{Z}^{k-1})$
- **filtering:** $p(\mathrm{x}_k|\mathcal{Z}^{k-1}) \xrightarrow[\text{sensor model}]{\text{sensor data } Z_k} p(\mathrm{x}_k|\mathcal{Z}^{k})$
- **retrodiction:** $p(\mathbf{x}_{l-1}|\mathcal{Z}^{k}) \xleftarrow[\text{dynamics model}]{\text{filtering output}} p(\mathbf{x}_l|\mathcal{Z}^{k})$

– ***finite mixture:*** **inherent ambiguity (data, model, road *network*)**
– ***optimal estimators:*** **e.g. minimum mean squared error (MMSE)**
– ***initiation of pdf iteration:*** **multiple hypothesis track extraction**

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Recapitulation: The Multivariate GAUSSian Pdf

– *wanted:* probabilities 'concentrated' around a center $\bar{\mathbf{x}}$

– *quadratic distance:* $q(\mathbf{x}) = \frac{1}{2}(\mathbf{x} - \bar{\mathbf{x}})\mathbf{P}^{-1}(\mathbf{x} - \bar{\mathbf{x}})^\top$

$q(\mathbf{x})$ defines an ellipsoid around $\bar{\mathbf{x}}$, its volume and orientation being determined by a matrix $\mathbf{P}$ (symmetric: $\mathbf{P}^\top = \mathbf{P}$, positively definite: all eigenvalues > 0).

– *first attempt:* $p(\mathbf{x}) = \mathrm{e}^{-q(\mathbf{x})} / \int d\mathbf{x}\; \mathrm{e}^{-q(\mathbf{x})}$ (normalized!)

$$p(\mathbf{x}) = \mathcal{N}(\mathbf{x};\, \bar{\mathbf{x}},\, \mathbf{P}) = \frac{1}{\sqrt{|2\pi\mathbf{P}|}}\, \mathrm{e}^{-\frac{1}{2}(\mathbf{x}-\bar{\mathbf{x}})^\top \mathbf{P}^{-1}(\mathbf{x}-\bar{\mathbf{x}})}$$

– GAUSS*ian Mixtures:* $p(\mathbf{x}) = \sum_i p_i\, \mathcal{N}(\mathbf{x};\, \bar{\mathbf{x}}_i,\, \mathbf{P}_i)$ (weighted sums)

Very First Look at an Important Data Fusion Algorithm

Kalman filter: $\mathbf{x}_k = (\mathbf{r}_k^\top, \dot{\mathbf{r}}_k^\top)^\top$, $\mathcal{Z}^k = \{\mathbf{z}_k, \mathcal{Z}^{k-1}\}$

initiation: $p(\mathbf{x}_0) = \mathcal{N}\big(\mathbf{x}_0;\, \mathbf{x}_{0|0},\, \mathbf{P}_{0|0}\big)$, initial ignorance: $\mathbf{P}_{0|0}$ 'large'

prediction: $\mathcal{N}\big(\mathbf{x}_{k-1};\, \mathbf{x}_{k-1|k-1},\, \mathbf{P}_{k-1|k-1}\big) \xrightarrow[\mathbf{F}_{k|k-1},\, \mathbf{D}_{k|k-1}]{\text{dynamics model}} \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{k|k-1},\, \mathbf{P}_{k|k-1}\big)$

$$\mathbf{x}_{k|k-1} = \mathbf{F}_{k|k-1}\mathbf{x}_{k-1|k-1}$$
$$\mathbf{P}_{k|k-1} = \mathbf{F}_{k|k-1}\mathbf{P}_{k-1|k-1}\mathbf{F}_{k|k-1}{}^\top + \mathbf{D}_{k|k-1}$$

filtering: $\mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{k|k-1},\, \mathbf{P}_{k|k-1}\big) \xrightarrow[\text{sensor model: } \mathbf{H}_k,\, \mathbf{R}_k]{\text{current measurement } \mathbf{z}_k} \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{k|k},\, \mathbf{P}_{k|k}\big)$

$$\mathbf{x}_{k|k} = \mathbf{x}_{k|k-1} + \mathbf{W}_{k|k-1}\boldsymbol{\nu}_{k|k-1}, \qquad \boldsymbol{\nu}_{k|k-1} = \mathbf{z}_k - \mathbf{H}_k\mathbf{x}_{k|k-1}$$
$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{W}_{k|k-1}\mathbf{S}_{k|k-1}\mathbf{W}_{k|k-1}{}^\top, \qquad \mathbf{S}_{k|k-1} = \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k{}^\top + \mathbf{R}_k$$
$$\mathbf{W}_{k|k-1} = \mathbf{P}_{k|k-1}\mathbf{H}_k{}^\top\mathbf{S}_{k|k-1}{}^{-1}$$
'KALMAN gain matrix'

A deeper look into the dynamics and sensor models necessary!

How to deal with probability density functions?

- pdf $p(x)$: Extract *probability statements* about the RV x by integration!
- naïvely: *positive* and *normalized* functions ($p(x) \geq 0$, $\int dx\, p(x) = 1$)
- *conditional pdf* $p(x|y) = \frac{p(x,y)}{p(y)}$: Impact of information on y on RV x?
- *marginal density* $p(x) = \int dy\, p(x,y) = \int dy\, p(x|y)\, p(y)$: Enter y!
- Bayes: $p(x|y) = \frac{p(y|x)p(x)}{p(y)} = \frac{p(y|x)p(x)}{\int dx\, p(y|x)p(x)}$: $p(x|y) \leftarrow p(y|x), p(x)$!

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- *certain knowledge* on x: $p(x) = \delta(x-y)$ '=' $\lim_{\sigma \to 0} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-y)^2}{\sigma^2}}$

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  $\int dx\, \delta(y - t[x])\, p_x(x) =: [\mathcal{T} p_x](y)$ ($\mathcal{T} : p_x \mapsto p$, "transfer operator")

Affine Transforms of GAUSSian Random Variables

$$\mathcal{N}\Big(\mathbf{x};\, \bar{\mathbf{x}},\, \mathbf{P}\Big) \xrightarrow{\mathbf{y}=\mathbf{t}+\mathbf{T}\mathbf{x}} \mathcal{N}\Big(\mathbf{y};\, \mathbf{t}+\mathbf{T}\bar{\mathbf{x}},\, \mathbf{T}\mathbf{P}\mathbf{T}^\top\Big)$$

$$p(\mathbf{y}) = \int d\mathbf{x}\, p(\mathbf{x},\mathbf{y}) = \int d\mathbf{x}\, p(\mathbf{y}|\mathbf{x})\, p(\mathbf{x}) = \int d\mathbf{x}\, \delta(\mathbf{y}-\mathbf{t}-\mathbf{T}\mathbf{x})\, p(\mathbf{x})$$

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A possible representation: $\delta(\mathbf{y}-\mathbf{t}-\mathbf{T}\mathbf{x}) = \mathcal{N}\big(\mathbf{y};\, \mathbf{t}+\mathbf{T}\mathbf{x},\, \mathbf{R}\big)$ with $\mathbf{R} \to \mathbf{O}$!

$$p(\mathbf{y}) = \int d\mathbf{x}\, \mathcal{N}(\mathbf{y}-\mathbf{t};\, \mathbf{T}\mathbf{x}, \mathbf{R})\, \mathcal{N}(\mathbf{x};\, \bar{\mathbf{x}}, \mathbf{P})$$

for $\mathbf{R} \to 0$ product formula!

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A Useful Product Formula for GAUSSians

$$\mathcal{N}(\mathbf{z};\, \mathbf{H}\mathbf{x},\, \mathbf{R})\; \mathcal{N}(\mathbf{x};\, \mathbf{y},\, \mathbf{P}) = \underbrace{\mathcal{N}(\mathbf{z};\, \mathbf{H}\mathbf{y},\, \mathbf{S})}_{\text{independent of } \mathbf{x}}\; \mathcal{N}(\mathbf{x};\, \mathbf{y} + \mathbf{W}\boldsymbol{\nu},\, \mathbf{P} - \mathbf{W}\mathbf{S}\mathbf{W}^\top)$$

$$\boldsymbol{\nu} = \mathbf{z} - \mathbf{H}\mathbf{y}, \quad \mathbf{S} = \mathbf{H}\mathbf{P}\mathbf{H}^\top + \mathbf{R}, \quad \mathbf{W} = \mathbf{P}\mathbf{H}^\top\mathbf{S}^{-1}.$$

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Affine Transforms of GAUSSian Random Variables

$$\boxed{\mathcal{N}\big(\mathbf{x};\, \bar{\mathbf{x}},\, \mathbf{P}\big) \xrightarrow{\mathbf{y}=\mathbf{t}+\mathbf{T}\mathbf{x}} \mathcal{N}\big(\mathbf{y};\, \mathbf{t}+\mathbf{T}\bar{\mathbf{x}},\, \mathbf{T}\mathbf{P}\mathbf{T}^\top\big)}$$

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$$p(\mathbf{y}) = \int d\mathbf{x}\, \mathcal{N}(\mathbf{y}-\mathbf{t};\mathbf{T}\mathbf{x},\mathbf{R})\, \mathcal{N}(\mathbf{x};\bar{\mathbf{x}},\mathbf{P}) \quad \text{for } \mathbf{R} \to 0$$

$$= \mathcal{N}\big(\mathbf{y};\, \mathbf{t}+\mathbf{T}\bar{\mathbf{x}},\, \mathbf{T}\mathbf{P}\mathbf{T}^\top + \mathbf{R}\big) \qquad \text{for } \mathbf{R} \to 0; \quad \text{product formula!}$$

Also true if $\dim(\mathbf{x}) \neq \dim(\mathbf{y})$**!**

Create your own sensor simulator!

Exercise 3.1

Simulate normally distributed (radar) measurements!

Measurement interval: ΔT = 5 s, sensor position: $\mathbf{r}_s$

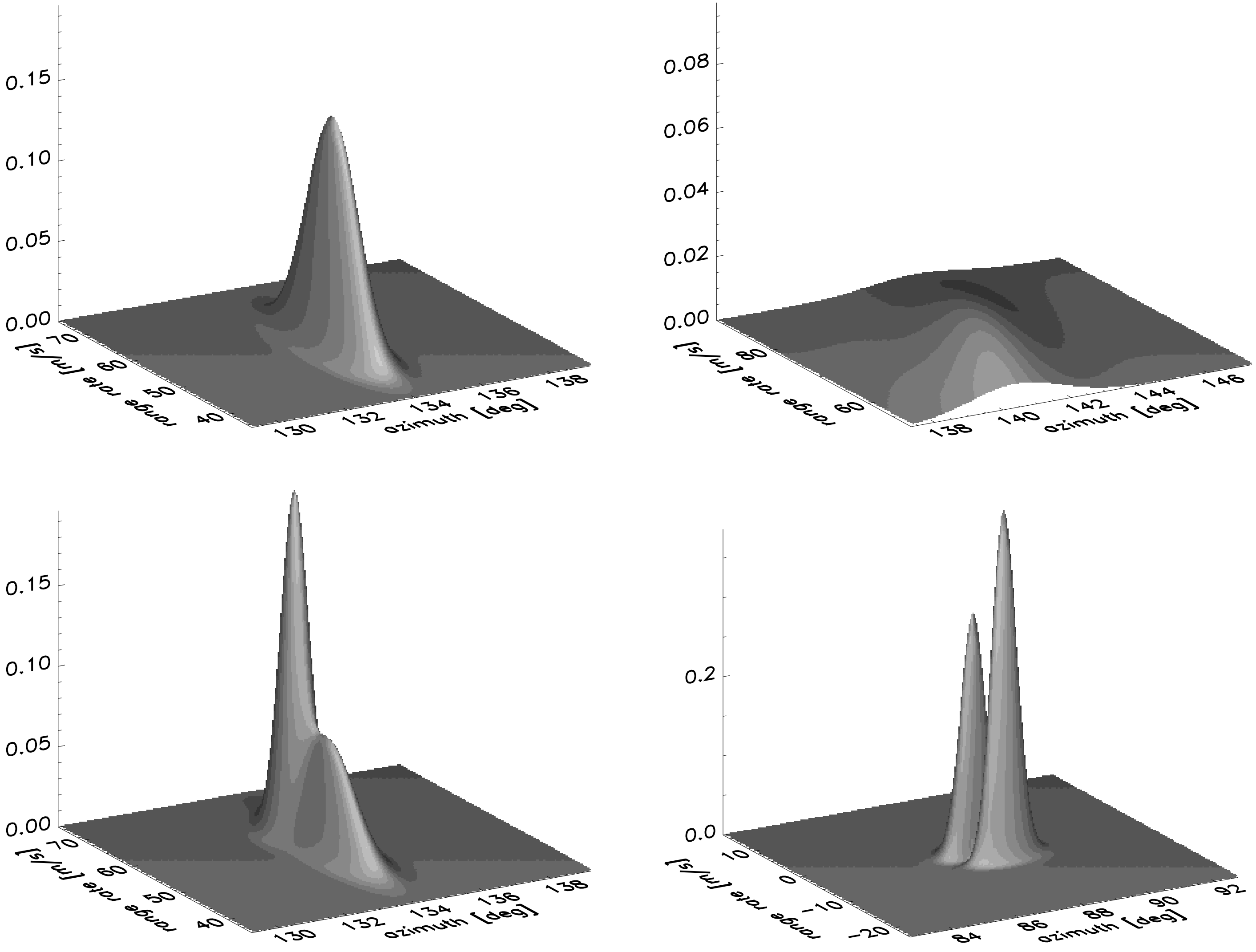
State at time $t_k = k\Delta T$, $k \in \mathbb{Z}$: $\mathbf{x}_k = (\mathbf{r}_k^\top, \dot{\mathbf{r}}_k^\top, \ddot{\mathbf{r}}_k^\top)^\top$

Use standard random number generators

such as normrnd(0, 1) that are producing

"normally distributed zero-mean, unit-variance random numbers"!

Interpret unknown object states as *random variables*, x [1D] or $\mathbf{x}$ [vector variate]), characterized by corresponding *probability density functions* (pdf).



The concrete shape of the pdf $p(\mathbf{x})$ contains the full knowledge on $\mathbf{x}$!

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Information on a random variable (RV) can be extracted by integration from the corresponding pdf!

at present: one dimensional case:

How probable is it that $x \in (a, b) \subseteq \mathbb{R}$ ***holds?***

Answer: $P\{x \in (a, b)\} = \int_a^b dx\, p(x) \quad \Rightarrow \quad p(x) \geq 0$

in particular: $P\{x \in \mathbb{R}\} = \int_{-\infty}^{\infty} dx\, p(x) = 1$ (normalzation)

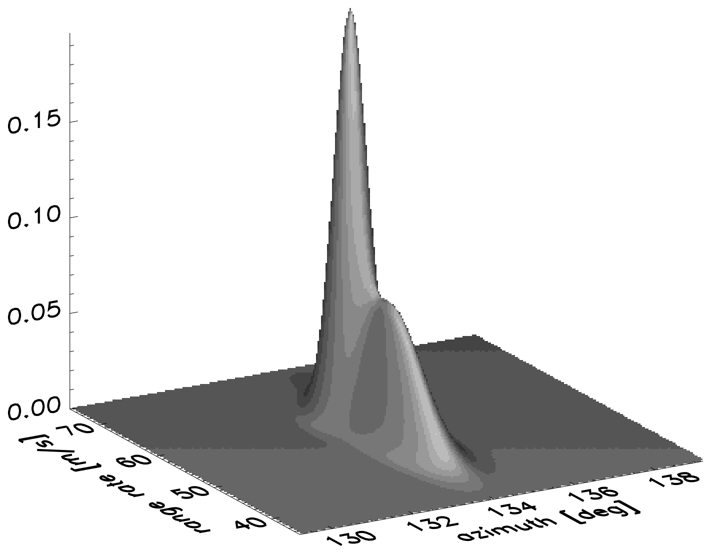
intuitive interpretation: "*the object is somewhere in* $\mathbb{R}$"

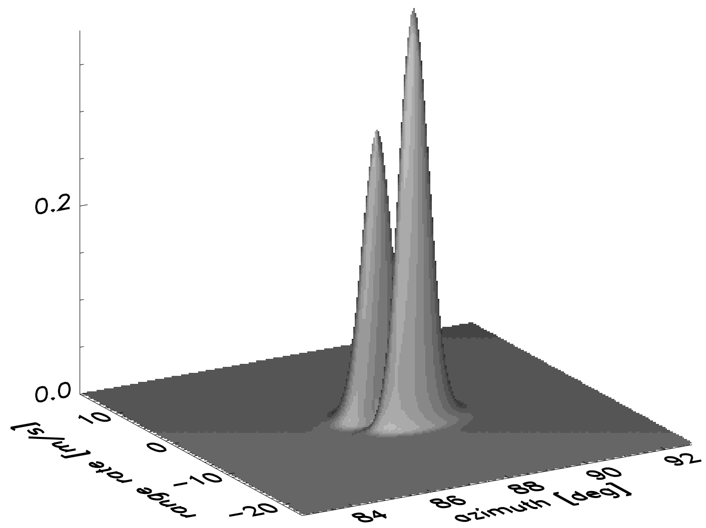
loosely: $p(x)\, dx$ **is probabity for** x **having a value between** x **and** $x + dx$

How to characterize the properties of a pdf?

specifically: How to associate a single "expected" value to a RV?

The maximum of the pdf is sometimes but not always useful!





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specifically: How to associate a single "expected" value to a RV?

The maximum of the pdf is sometimes but not always useful! ($\rightarrow$ examples)

instead: Calculate the centroid of the pdf!

$$\mathbb{E}[x] = \int_{-\infty}^{\infty} dx \, x \, p(x) = \bar{x} \quad \text{"expectation value"}$$

How to characterize the properties of a pdf?

specifically: How to associate a single “expected” value to a RV?

The maximum of the pdf is sometimes but not always useful! ($\rightarrow$ examples)

instead: Calculate the centroid of the pdf!

$$\mathbb{E}[x] = \int_{-\infty}^{\infty} dx\, x\, p(x) = \bar{x}$$ “expectation value”

more generally: Consider functions $g : x \mapsto g(x)$ of the RV x!

$$\mathbb{E}[g(x)] = \int_{-\infty}^{\infty} dx\, g(x)\, p(x),$$ “expectation value of the *observable* g“

Example: Consider the observable $\frac{1}{2}mx^2$ (kinetic energy, x = speed)

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An important observable: the "error" of an estimate

- **Quality:** How useful is an expectation value $\bar{x} = \mathbb{E}[x]$?

Consider special obervables as distance measure:

$$g(x) = |x - \bar{x}| \quad \text{oder} \quad g(x) = (x - \bar{x})^2$$

quadratic measures: computationally more comfortable!

'expected error' of the expectation value $\bar{x}$:

$$\boxed{\mathbb{V}[x] = \mathbb{E}[(x - \bar{x})^2],\ \sigma_x = \sqrt{\mathbb{V}[x]}}$$

variance, standard deviation

Exercise 3.2 Show that $\boxed{\mathbb{V}[x] = \mathbb{E}[x^2] - \mathbb{E}[x]^2}$ holds.

Expectation value of the observable x^2 also called "2nd moment" of the pdf of x.

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Calculate expectation and variance of the uniform density
of a RV $x \in \mathbb{R}$ in the intervall $[a, b]$.

Exercise 3.3

$$p(x) = \mathcal{U}(\underbrace{x}_{\text{ZV}};\ \underbrace{a, b}_{\text{Parameter}}) = \begin{cases} \frac{1}{b-a} & x \in [a, b] \\ 0 & \text{sonst} \end{cases}$$

!

Pdf correctly normalized? $\int_{-\infty}^{\infty} dx\, \mathcal{U}(x; a, b) = \frac{1}{b-a} \int_a^b dx = 1$

$$\mathbb{E}[x] = \int_{-\infty}^{\infty} dx\, x\, \mathcal{U}(x; a, b) = \frac{b+a}{2}$$

$$\mathbb{V}[x] = \frac{1}{b-a} \int_a^b dx\, x^2 - \mathbb{E}[x]^2 = \frac{1}{12}(b-a)^2$$

Important example: x *normally distributed* over $\mathbb{R}$ (Gauss)

- *wanted:* probabilities concentrated around μ
- quadratic distance: $||x-\mu||^2 = \frac{1}{2}(x-\mu)^2/\sigma^2$ (mathematically convenient!)
- Parameter σ is a measure of the "width" of the pdf: $||\sigma||^2 = \frac{1}{2}$
- for 'large' distances, i.e. $||x-\mu||^2 \gg \frac{1}{2}$, the pdf shall decay quickly.
- simplest approach: $\tilde{p}(x) = \mathrm{e}^{-||x-\mu||^2}$ $\quad$ ($> 0\ \forall x \in \mathbb{R}$, normalization?)
- Normalized for $p(x) = \tilde{p}(x)/\int_{-\infty}^{\infty} dx\, \tilde{p}(x)$!

Formula collection delivers: $\displaystyle\int_{-\infty}^{\infty} dx\, \tilde{p}(x) = \sqrt{2\pi}\sigma$

An admissible pdf with the required properties is obviously given by:

$$\boxed{\mathcal{N}(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\, \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

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Exercise 3.4

Show for the Gauss density $p(x) = \mathcal{N}(x; \mu, \sigma)$:

$$\boxed{\mathbb{E}[x] = \mu, \ \mathbb{V}[x] = \sigma^2}$$

$$\mathbb{E}[x] = \int_{-\infty}^{\infty} dx \, x \, \mathcal{N}(x; \mu, \sigma) = \mu$$

$$\mathbb{V}[x] = \mathbb{E}[x^2] - \mathbb{E}[x]^2 = \sigma^2$$

Use substitution and partial integration!

Use $\int_{-\infty}^{\infty} dx \, \mathrm{e}^{-\frac{1}{2}x^2} = \sqrt{2\pi}$!

Create your own sensor simulator!

Exercise 3.1

Simulate normally distributed (radar) measurements!

Measurement interval: ΔT = 5 s, sensor position: $\mathbf{r}_s$

State at time $t_k = k\Delta T$, $k \in \mathbb{Z}$: $\mathbf{x}_k = (\mathbf{r}_k^\top, \dot{\mathbf{r}}_k^\top, \ddot{\mathbf{r}}_k^\top)^\top$

Use standard random number generators such as normrnd(0, 1)!

$$\text{with } \bar{\mathbf{u}}_k = \begin{pmatrix} \text{normrnd}(0,1) \\ \text{normrnd}(0,1) \end{pmatrix} : \quad p(\bar{\mathbf{u}}_k) = \mathcal{N}\big(\bar{\mathbf{u}}_k;\, \mathbf{o},\, \mathbf{I}\big)$$

$$\text{we have for } \mathbf{u}_k = \sigma\bar{\mathbf{u}}_k : \quad p(\mathbf{u}_k) = \mathcal{N}\big(\mathbf{u}_k;\, \mathbf{o},\, \sigma^2\mathbf{I}\big)$$

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1. Simulate measurements of the Cartesian position components of the target state $\mathbf{x}_k$:

$$\mathbf{z}_k^c = \begin{pmatrix} z_k^x \\ z_k^y \end{pmatrix} = \mathbf{H}\mathbf{x}_k + \mathbf{u}_k = (\,\mathrm{I}\;\mathrm{I}\;\mathrm{I}\,)\begin{pmatrix} \mathbf{r}_k \\ \dot{\mathbf{r}}_k \\ \ddot{\mathbf{r}}_k \end{pmatrix} + \sigma_c \begin{pmatrix} \text{normrnd}(0,1) \\ \text{normrnd}(0,1) \end{pmatrix}$$

with a random number generator normrnd$(0, 1)$ producing normally distributed zero-mean and unit-variance random numbers, σ_c = 50 m denoting the standard deviation of the sensor measurement errors. Sensor position has no impact.

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2. Simulate range / azimuth measurements of the target position $\mathbf{r}_k$ w.r.t sensor position $\mathbf{r}_s$:

$$\mathbf{z}_k^p = \begin{pmatrix} z_k^r \\ z_k^\varphi \end{pmatrix} = \begin{pmatrix} \sqrt{(x_k - x_s)^2 + (y_k - y_s)^2} \\ \arctan(\frac{y_k - y_s}{x_k - x_s}) \end{pmatrix} + \begin{pmatrix} \sigma_r \,\text{normrnd}(0,1) \\ \sigma_\varphi \,\text{normrnd}(0,1) \end{pmatrix}, \quad \mathbf{r}_{k,s} = (x_{k,s}, y_{k,s})^\top$$

   with σ_r = 20 m, σ_φ = 0.2° denoting the standard deviations in range and azimuth.

3. Plot the Cartesian and polar measurements $z_k^r(\cos z_k^\varphi, \sin z_k^\varphi)^\top + \mathbf{r}_s$ over the true target trajectory! Play with sensor positions and measurement error standard deviations!

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Read (if you like) the Wikipedia article on John von Neumann's (1903-1957) algorithm for "rejection sampling"
http://en.wikipedia.org/wiki/Rejection_sampling.

Generate (if you like) random numbers z_n with $p(z_n) = \mathcal{N}(z_n; 0, 1)$ from random numbers z_u with $p(z_u) = \mathcal{U}(z_u; 0, 1)$.

Read the Wikipedia article on the great mathematician, physicist and computer pioneer John von Neumann
http://en.wikipedia.org/wiki/John_von_Neumann.

Exercise 3.5

Read the beautiful book (if you like): *George Dyson (2012). Turing's Cathedral: The Origins of the Digital Universe*.

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Generalization to multiple random variables!

Vector states: $\mathbf{x} = (x_1, \ldots, x_{n-1}, x_n)^\top$ e.g. : $\mathbf{x} = (\mathbf{r}, \dot{\mathbf{r}})^\top, \mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2)^\top$

Volume integral: $P\{\mathbf{x} \in V\} = \int_V dx_1 \ldots dx_n \, p(x_1, \ldots, x_n)$

vector variate or scalar expectation values: $\mathbb{E}[g(\mathbf{x})] = \int d\mathbf{x} \, g(\mathbf{x}) \, p(\mathbf{x})$

Independence: Statements about x not influenced by $y \rightarrow \; p(x, y) = p(x) \, p(y)$!

$$P\{x \in X, y \in Y\} = \int_X dx \ldots dx \, p(x) \, \int_Y dy \ldots dy \, p(y)$$

Some properties of joint densities

Non-negative: $p(x, y) \geq 0$

Normalized: $\int dx\, dy\, p(x, y) = 1$

Relation between $p(x)$, $p(y)$ and $p(x, y)$:

$$p(x) = \int dy\, p(x, y)$$

$$p(y) = \int dx\, p(x, y)$$

$p(x)$ is also called a marginal density of the joint density w.r.t. x.

How does knowledge on a RV y affects knowledge on a RV x?

No impact if x, y are independent of each other. $\rightarrow$ $p(x|y) = p(x)$

Feeling: $p(x, y)$ and $p(y)$ should enter into the definition of $p(x|y)$.

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A first attempt: $$\boxed{p(x|y) = \frac{p(x, y)}{p(y)}}$$

- $\int dx\, p(x|y) = \frac{1}{p(y)} \int dx\, p(x, y) = \frac{p(y)}{p(y)} = 1$ $\quad\rightarrow$ Normalized!
- x, y mutually independent: $p(x|y) = \frac{p(x, y)}{p(y)} = \frac{p(x)\, p(y)}{p(y)} = p(x)$

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- x, y mutually independent: $p(x|y) = \frac{p(x,y)}{p(y)} = \frac{p(x)\, p(y)}{p(y)} = p(x)$

$p(x|y) \geq 0$ is obviously interpretable as a useful pdf that quantitatively describes the notions of statistical "dependency" and "independency".

conditional probability density function: $p(x|y)$

How to calculate conditional pdfs? Use Bayes' Rule!

Because of: $$p(x|y)\, p(y) = p(x,y) = p(y,x) = p(y|x)\, p(x)$$

we have in particular: $$p(x|y) = \frac{p(y|x)\, p(x)}{p(y)}$$

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we have in particular: $p(x|y) = \dfrac{p(y|x)\,p(x)}{p(y)}$

We can also write: $p(y) = \underbrace{\int dx\, p(y,x)}_{\text{marginal pdf}} = \int dx\ \underbrace{p(y|x)\,p(x)}_{\text{def. cond. pdf}}$

and thus obtain:

$$\boxed{p(x|y) = \frac{p(y|x)\,p(x)}{\int dx\, p(y|x)\,p(x)}}$$

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and thus obtain: $$\boxed{p(x|y) = \frac{p(y|x)\,p(x)}{\int dx\, p(y|x)\,p(x)}}$$

- Who knows $p(y|x)$ and $p(x)$, can calculate how knowledge on y affects knowledge on x.
- Large parts of statistics is just an application of Bayes' rule. (Rev. Thomas Bayes, 18th century, fully understood by Laplace)

REV. T. BAYES

Google Rev. Thomas Bayes (1701-1761), perhaps starting with

`http://en.wikipedia.org/wiki/Thomas_Bayes`.

More Precise Formulation of the BAYESian Approach

Consider a set of measurements $Z_l = \{\mathbf{z}_l^j\}_{j=1}^{m_l}$ of a single or a multiple target state $\mathbf{x}_l$ at time instants t_l, $l = 1, \ldots, k$ and the time series:

$$\mathcal{Z}^k = \{Z_k, m_k, Z_{k-1}, m_{k-1}, \ldots, Z_1, m_1\} = \{Z_k, m_k, \mathcal{Z}^{k-1}\}!$$

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$$\mathcal{Z}^k = \{Z_k, m_k, Z_{k-1}, m_{k-1}, \dots, Z_1, m_1\} = \{Z_k, m_k, \mathcal{Z}^{k-1}\}!$$

Based on $\mathcal{Z}^k$, what can be learned about the object states $\mathbf{x}_l$ at $t_1, \dots, t_k, t_{k+1}, \dots$, i.e. for the past, present, and future?

Evidently the answer is given be calculating the pdf $p(\mathbf{x}_l|\mathcal{Z}^k)$!

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multiple sensor measurement fusion: Calculate $p(\mathbf{x}|\mathcal{Z}_1^k, \dots, \mathcal{Z}_N^k)$!

- communication lines
- common coordinate system: sensor registration

How to calculate the pdf $p(\mathrm{x}_l|\mathcal{Z}^k)$?

Consider at first the present time: $l = k$.

an observation:

Bayes' rule: $p(\mathrm{x}_k|\mathcal{Z}^k) = p(\mathrm{x}_k|Z_k, m_k, \mathcal{Z}^{k-1})$

$$= \frac{p(Z_k, m_k|\mathrm{x}_k, \mathcal{Z}^{k-1})\, p(\mathrm{x}_k|\mathcal{Z}^{k-1})}{\int d\mathrm{x}_k\, p(Z_k, m_k|\mathrm{x}_k, \mathcal{Z}^{k-1})\, p(\mathrm{x}_k|\mathcal{Z}^{k-1})}$$

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$$= \frac{p(Z_k, m_k|\mathrm{x}_k, \mathcal{Z}^{k-1})\, p(\mathrm{x}_k|\mathcal{Z}^{k-1})}{\int d\mathrm{x}_k\ \underbrace{p(Z_k, m_k|\mathrm{x}_k, \mathcal{Z}^{k-1})}_{\text{likelihood function}}\ \underbrace{p(\mathrm{x}_k|\mathcal{Z}^{k-1})}_{\text{prediction}}}$$

- $p(\mathrm{x}_k|\mathcal{Z}^{k-1})$ is a *prediction* of the target state at time t_k based on all measurements in the *past*.

- $p(Z_k, m_k|\mathrm{x}_k) \propto \ell(\mathrm{x}_k; Z_k, m_k)$ describes, what the *current* sensor output Z_k, m_k can say about the current target state x_k and is called *likelihood function*.

- $p(\mathbf{x}_k|\mathcal{Z}^{k-1})$ is a *prediction* for time t_k based on all measurements in the *past*.

$$p(\mathbf{x}_k|\mathcal{Z}^{k-1}) = \int d\mathbf{x}_{k-1}\, p(\mathbf{x}_k, \mathbf{x}_{k-1}|\mathcal{Z}^{k-1}) \quad \text{marginal pdf}$$

$$= \int d\mathbf{x}_{k-1}\, \underbrace{p(\mathbf{x}_k|\mathbf{x}_{k-1}, \mathcal{Z}^{k-1})}_{\text{object dynamics!}}\, \underbrace{p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1})}_{\text{idea: iteration!}} \quad \text{notion of a conditional pdf}$$

sometimes: $p(\mathbf{x}_k|\mathbf{x}_{k-1}) = \mathcal{N}\big(\mathbf{x}_k;\, \underbrace{\mathbf{F}_{k|k-1}}_{\text{deterministic}}\mathbf{x}_{k-1},\, \underbrace{\mathbf{D}_{k|k-1}}_{\text{random}}\big)$ (linear GAUSS-MARKOV)

- $p(Z_k, m_k|\mathbf{x}_k) \propto \ell(\mathbf{x}_k; Z_k, m_k)$ describes, what the *current* sensor output Z_k, m_k can say about the current target state $\mathbf{x}_k$ and is called *likelihood function*.

sometimes: $\ell(\mathbf{x}_k; \mathbf{z}_k) = \mathcal{N}\big(\mathbf{z}_k;\, \mathbf{H}_k\mathbf{x}_k,\, \mathbf{R}_k\big)$ (1 target, 1 measurement)

iteration formula: $$p(\mathbf{x}_k|\mathcal{Z}^k) = \frac{\ell(\mathbf{x}_k; \mathbf{z}_k) \int d\mathbf{x}_{k-1}\, p(\mathbf{x}_k|\mathbf{x}_{k-1})\, p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1})}{\int d\mathbf{x}_k\, \ell(\mathbf{x}_k; \mathbf{z}_k) \int d\mathbf{x}_{k-1}\, p(\mathbf{x}_k|\mathbf{x}_{k-1})\, p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1})}$$

A popular model for object evolutions

Piecewise Constant White Acceleration Model

Consider state vectors: $\mathbf{x}_k = (\mathbf{r}_k^\top, \dot{\mathbf{r}}_k^\top)^\top$ (position, velocity)

For known $\mathbf{x}_{k-1}$ and without external influences we have with $\Delta T_k = t_k - t_{k-1}$:

$$\mathbf{x}_k = \begin{pmatrix} \mathbf{I} & \Delta T_k \, \mathbf{I} \\ \mathbf{O} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{r}_{k-1} \\ \dot{\mathbf{r}}_{k-1} \end{pmatrix} =: \mathbf{F}_{k|k-1} \mathbf{x}_{k-1},$$ **see blackboard!**

Assume during the interval ΔT_k a constant acceleration $\mathbf{a}_k$ causing the state evolution:

$$\begin{pmatrix} \frac{1}{2} \Delta T_k^2 \, \mathbf{I} \\ \Delta T_k \, \mathbf{I} \end{pmatrix} \mathbf{a}_k =: \mathbf{G}_k \mathbf{a}_k,$$ **linear transform!**

Let $\mathbf{a}_k$ be a Gaussian RV with pdf: $p(\mathbf{a}_k) = \mathcal{N}\big(\mathbf{a}_k;\, \mathbf{o},\, \Sigma_k^2 \mathbf{I}\big)$, we therefore have:

$$p(\mathbf{G}_k \mathbf{a}_k) = \mathcal{N}\big(\mathbf{G}_k \mathbf{a}_k;\, \mathbf{o},\, \Sigma_k^2 \mathbf{G}_k \mathbf{G}_k^\top\big).$$

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Therefore: $p(\mathbf{x}_k|\mathbf{x}_{k-1}) = \mathcal{N}\left(\mathbf{x}_k;\, \mathbf{F}_{k|k-1}\mathbf{x}_{k-1},\, \mathbf{D}_{k|k-1}\right)$ with

$$\mathbf{F}_{k|k-1} = \begin{pmatrix} \mathbf{I} & \Delta T_k\,\mathbf{I} \\ \mathbf{O} & \mathbf{I} \end{pmatrix}, \quad \mathbf{D}_{k|k-1} = \Sigma_k^2 \begin{pmatrix} \frac{1}{4}\Delta T_k^4\,\mathbf{I} & \frac{1}{2}\Delta T^3\,\mathbf{I} \\ \frac{1}{2}\Delta T_k^3\,\mathbf{I} & \Delta T_k^2\,\mathbf{I} \end{pmatrix}$$

Therefore: $\quad p(\mathbf{x}_k|\mathbf{x}_{k-1}) = \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{F}_{k|k-1}\mathbf{x}_{k-1},\, \mathbf{D}_{k|k-1}\big)$ with

$$\mathbf{F}_{k|k-1} = \begin{pmatrix} \mathbf{I} & \Delta T_k\, \mathbf{I} \\ \mathbf{O} & \mathbf{I} \end{pmatrix}, \quad \mathbf{D}_{k|k-1} = \Sigma_k^2 \begin{pmatrix} \frac{1}{4}\Delta T_k^4\, \mathbf{I} & \frac{1}{2}\Delta T_k^3\, \mathbf{I} \\ \frac{1}{2}\Delta T_k^3\, \mathbf{I} & \Delta T_k^2\, \mathbf{I} \end{pmatrix}$$

Exercise 3.6 Consider $\mathbf{x}_k = (\mathbf{r}_k^\top, \dot{\mathbf{r}}_k^\top, \ddot{\mathbf{r}}_k^\top)^\top$ (position, velocity, acceleration)

Show that $\mathbf{F}_{k|k-1}$ and $\mathbf{D}_{k|k-1} = \Sigma_k^2 \mathbf{G}_k \mathbf{G}_k^\top$ (constant acceleration rates) are given by:

$$\mathbf{F}_{k|k-1} = \begin{pmatrix} \mathbf{I} & \Delta T_k\, \mathbf{I} & \frac{1}{2}\Delta T_k^2\, \mathbf{I} \\ \mathbf{O} & \mathbf{I} & \Delta T_k\, \mathbf{I} \\ \mathbf{O} & \mathbf{I} & \mathbf{I} \end{pmatrix}, \quad \mathbf{D}_{k|k-1} = \Sigma_k^2 \begin{pmatrix} \frac{1}{4}\Delta T_k^4\, \mathbf{I} & \frac{1}{2}\Delta T_k^3\, \mathbf{I} & \frac{1}{2}\Delta T_k^2\, \mathbf{I} \\ \frac{1}{2}\Delta T_k^3\, \mathbf{I} & \Delta T_k^2 \mathbf{I} & \Delta T_k\, \mathbf{I} \\ \frac{1}{2}\Delta T_k^2\, \mathbf{I} & \Delta T_k \mathbf{I} & \mathbf{I} \end{pmatrix}$$

with $\Delta T_k = t_k - t_{k-1}$. Reasonable choice: $\frac{1}{2}q_{\text{max}} \le \Sigma_k \le q_{\text{max}}$

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A more Insightful Look at a Data Fusion Algorithm

Kalman filter: $\mathbf{x}_k = (\mathbf{r}_k^\top, \dot{\mathbf{r}}_k^\top)^\top$, $\mathcal{Z}^k = \{\mathbf{z}_k, \mathcal{Z}^{k-1}\}$

initiation: $p(\mathbf{x}_0) = \mathcal{N}\big(\mathbf{x}_0;\, \mathbf{x}_{0|0},\, \mathbf{P}_{0|0}\big)$, initial ignorance: $\mathbf{P}_{0|0}$ 'large'

prediction: $\mathcal{N}\big(\mathbf{x}_{k-1};\, \mathbf{x}_{k-1|k-1},\, \mathbf{P}_{k-1|k-1}\big) \xrightarrow[\mathbf{F}_{k|k-1},\, \mathbf{D}_{k|k-1}]{\text{dynamics model}} \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{k|k-1},\, \mathbf{P}_{k|k-1}\big)$

$$\mathbf{x}_{k|k-1} = \mathbf{F}_{k|k-1}\mathbf{x}_{k-1|k-1}$$
$$\mathbf{P}_{k|k-1} = \mathbf{F}_{k|k-1}\mathbf{P}_{k-1|k-1}\mathbf{F}_{k|k-1}{}^\top + \mathbf{D}_{k|k-1}$$

filtering: $\mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{k|k-1},\, \mathbf{P}_{k|k-1}\big) \xrightarrow[\text{sensor model: } \mathbf{H}_k,\, \mathbf{R}_k]{\text{current measurement } \mathbf{z}_k} \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{k|k},\, \mathbf{P}_{k|k}\big)$

$$\mathbf{x}_{k|k} = \mathbf{x}_{k|k-1} + \mathbf{W}_{k|k-1}\boldsymbol{\nu}_{k|k-1}, \qquad \boldsymbol{\nu}_{k|k-1} = \mathbf{z}_k - \mathbf{H}_k\mathbf{x}_{k|k-1}$$
$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{W}_{k|k-1}\mathbf{S}_{k|k-1}\mathbf{W}_{k|k-1}{}^\top, \qquad \mathbf{S}_{k|k-1} = \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k{}^\top + \mathbf{R}_k$$
$$\mathbf{W}_{k|k-1} = \mathbf{P}_{k|k-1}\mathbf{H}_k{}^\top\mathbf{S}_{k|k-1}{}^{-1} \qquad \text{'KALMAN gain matrix'}$$

A deeper look into the dynamics and sensor models necessary!

Definitions: “Sensor Data and Information Fusion”

Llinas (2001). “Information fusion is an Information Process dealing with the association, correlation, and combination of data and information from single and multiple sensors or sources to achieve refined estimates of parameters, characteristics, events, and behaviors for observed entities in an observed field of view. It is sometimes implemented as a Fully Automatic process or as a Human-Aiding process for Analysis and/or Decision Support.” [1]

— —

JDL (1987). Data fusion is “a process dealing with the association, correlation, and combination of data and information from single and multiple sources to achieve refined position and identity estimates, and complete and timely assessments of situations and threats, and their significance. The process is characterized by continuous refinements of its estimates and assessments, and the evaluation of the need for additional sources, or modification of the process itself, to achieve improved results.” [2]

Hugh Durrant-Whyte (1988). “The basic problem in multi-sensor systems is to integrate a sequence of observations from a number of different sensors into a single best-estimate of the state of the environment.” [3]

Llinas (1988). “Fusion can be defined as a process of integrating information from multiple sources to produce the most specific and comprehensive unified data about an entity, activity or event. This definition has some key operative words: specific, comprehensive, and entity. From an informationtheoretic point of view, fusion, to be effective as an information processing function, must (at least ideally) increase the specificity and comprehensiveness of the understanding we have about a battlefield entity or else there would be no purpose in performing the function.” [4]

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Richardson and Marsh (1988). “Data fusion is the process by which data from a multitude of sensors is used to yield an optimal estimate of a specified state vector pertaining to the observed system.” [5]

McKendall and Mintz (1988). “...the problem of sensor fusion is the problem of combining multiple measurements from sensors into a single measurement of the sensed object or attribute, called the parameter.” [6]

Waltz and Llinas (1990). “This field of technology has been appropriately termed data fusion because the objective of its processes is to combine elements of raw data from different sources into a single set of meaningful information that is of greater benefit than the sum of the contributing parts. As a technology, data fusion is actually the integration and application of many traditional disciplines and new areas of engineering to achieve the fusion of data.” [7]

Luo and Kay (1992). “Multisensor fusion, ..., refers to any stage in an integration process where there is an actual combination (or fusion) of different sources of sensory information into one representational format.” [8]

Abidi and Gonzalez (1992). “Data fusion deals with the synergistic combination of information made available by various knowledge sources such as sensors, in order to provide a better understanding of a given scene.” [9]

Hall (1992). “Multisensor data fusion seeks to combine data from multiple sensors to perform inferences that may not be possible from a single sensor alone.” [10]

DSTO (1994). Data fusion is “a multilevel, multifaceted process dealing with the automatic detection, association, correlation, estimation, and combination of data and information from single and multiple sources.” [11]

Malhotra (1995). “The process of sensor fusion involves gathering sensory data, refining and interpreting it, and making new sensor allocation decisions.” [12]

Hall and Llinas (1997). “Data fusion techniques combine data from multiple sensors, and related information from associated databases, to achieve improved accuracy and more specific inferences than could be achieved by the use of single sensor alone.” [13]

Goodman, Mahler and Nguyen (1997). Data fusion is to “locate and identify many unknown objects of many different types on the basis of different kinds of evidence. This evidence is collected on an ongoing basis by many possibly allocatable sensors having varying capabilities and to analyze the results insuch a way as to supply local and over-all assessments of the significance of a scenario and to determine proper responses based on those assessments.” [14]

Paradis, Chalmers, Carling and Bergeron (1997). “Data fusion is fundamentally a process designed to manage (i.e., organize, combine and interpret) data and information, obtained from a variety of sources, that may be required at any time by operators or commanders for decision making. ... data fusion is an adaptive information process that continuously transforms available data and information into richer information, through continuous refinement of hypotheses or inferences about real-world events, to achieve a refined (potentially optimal) kinematics and identity estimates of individual objects, and complete and timely assessments of current and potential future situations and threats (i.e., contextual reasoning), and their significance in the context of operational settings.” [15]

Starr and Desforges (1998). “Data fusion is a process that combines data and knowledge from different sources with the aim of maximising the useful information content, for improved reliability or discriminant capability, while minimising the quantity of data ultimately retained.” [16]

Wald (1998). “Data fusion is a formal framework in which are expressed means and tools for the alliance of data of the same scene originating from different sources. It aims at obtaining information of greater quality; the exact definition of greater quality will depend upon the application.” [17]

Evans (1998). “The combining of data from different complementary sources (usually geodemographic and lifestyle or market research and lifestyle) to build a picture of someone’s life”. [18]

Wald (1999). “Data fusion is a formal framework in which are expressed the means and tools for the alliance of data originating from different sources.” [19]

Steinberg, Bowman and White (1999). “Data fusion is the process of combining data to refine state estimates and predictions.” [20]

Gonsalves, Cunningham, Ton and Okon (2000). “The overall goal of data fusion is to combine data from multiple sources into information that has greater benefit than what would have been derived from each of the contributing parts.” [21]

Hannah, Ball and Starr (2000). “Fusion is defined materially as a process of blending, usually with the application of heat to melt constituents together (OED), but in data processing the more abstract form of union or blending together is meant. The ’heat’ is applied with a series of algorithms which, depending on the technique used, give a more or less abstract relationship between the constituents and the finished output.” [22]

Dasarathy (2001). “Information fusion encompasses the theory, techniques, and tools conceived and employed for exploiting the synergy in the information acquired from multiple sources (sensor, databases, information gathered by humans etc.) such that the resulting decision or action is in some sense better (qualitatively and quantitatively, in terms of accuracy, robustness and etc.) than would be possible, if these sources were used individually without such synergy exploitation.” [23]

Bloch and Hunter et al. (2001). "...fusion consists in conjoining or merging information that stems from several sources and exploiting that conjoined or merged information in various tasks such as answering questions, making decisions, numerical estimation, etc." [24]

McGirr (2001). "The process of bringing large amounts of dissimilar information together into a more comprehensive and easily manageable form is known as data fusion." [25]

Bell, Santos and Brown (2002). "Sophisticated information fusion capabilities are required in order to transform what the agents gather from a raw form to an integrated, consistent and complete form. Information fusion can occur at multiple levels of abstraction." [26]

Challa, Gulrez, Chaczko and Paranesha (2005). Multi-sensor data fusion "is a core component of all networked sensing systems, which is used either to:- join/combine complementary information produced by sensor to obtain a more complete picture or - reduce/manage uncertainty by using sensor information from multiple sources." [27]

Jalobeanu and Gutirrez (2006). "The data fusion problem can be stated as the computation of the posterior pdf [probability distribution function] of the unknown single object given all observations." [28]

Mastrogiovanni et al (2007). "The aim of a data fusion process is to maximize the useful information content acquired by heterogeneous sources in order to infer relevant situations and events related to the observed environment." [29]

Wikipedia (2007). "Information Integration is a field of study known by various terms: Information Fusion, Deduplication, Referential Integrity and so on. It refers to the field of study of techniques attempting to merge information from disparate sources despite differing conceptual, contextual and typographical representations. This is used in data mining and consolidation of data from semi- or unstructured resources." [30]

Wikipedia (2007). “Sensor fusion is the combining of sensory data or data derived from sensory data from disparate sources such that the resulting information is in some sense better than would be possible when these sources were used individually. The term better in that case can mean more accurate, more complete, or more dependable, or refer to the result of an emerging view, such as stereoscopic vision (calculation of depth information by combining two-dimensional images from two cameras at slightly different viewpoints). The data sources for a fusion process are not specified to originate from identical sensors. One can distinguish direct fusion, indirect fusion and fusion of the outputs of the former two. Direct fusion is the fusion of sensor data from a set of heterogeneous or homogeneous sensors, soft sensors, and history values of sensor data, while indirect fusion uses information sources like a priori knowledge about the environment and human input. Sensor fusion is also known as (multi-sensor) data fusion and is a subset of information fusion.” [31]

MSN Encarta (2007). “Data integration: the integration of data and knowledge collected from disparate sources by different methods into a consistent, accurate, and useful whole.” [32]

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