Create your own ground truth generator!

Exercise 3.1

Consider a car moving on a mountain pass road modeled by:

$$\mathbf{r}(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} vt \\ a_y \sin(\frac{4\pi v}{a_x} t) \\ a_z \sin(\frac{\pi v}{a_x} t) \end{pmatrix}$$

$v = 20 \frac{\text{km}}{\text{h}}$, a_x = 10 km, $a_y = a_z = 1$ km, $t \in [0, a_x/v]$.

1. Plot the trajectory. Are the parameters reasonable? Try alternatives.
2. Calculate and plot the velocity and acceleration vectors:

$$\dot{\mathbf{r}}(t) = \begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \end{pmatrix}, \quad \ddot{\mathbf{r}}(t) = \begin{pmatrix} \ddot{x}(t) \\ \ddot{y}(t) \\ \ddot{z}(t) \end{pmatrix}.$$

3. Calculate for each instance of time t the tangential vectors in $\mathbf{r}(t)$:

$$\mathbf{t}(t) = \frac{1}{|\dot{\mathbf{r}}(t)|} \dot{\mathbf{r}}(t).$$

4. Plot $|\dot{\mathbf{r}}(t)|$, $|\ddot{\mathbf{r}}(t)|$, and $\ddot{\mathbf{r}}(t)\mathbf{t}(t)$ over the time interval.
5. Discuss the temporal behaviour based on the trajectory $\mathbf{r}(t)$!

Fraunhofer FKIE

Create your own sensor simulator!

Exercise 4.1

Simulate normally distributed radar measurements!

ΔT = 2 s, 2 radars at $\mathbf{r}_s^{1,2} = (x_s^{1,2}, y_s^{1,2}, z_s^{1,2})^\top$,

$x_s^{1,2}$ = 0, 100 km, $y_s^{1,2}$ = 100, 0 km, $z_s^{1,2}$ = 10 km.

State at time $t_k = k\Delta T$, $k \in \mathbb{Z}$: $\mathbf{x}_k = (\mathbf{r}_k^\top, \dot{\mathbf{r}}_k^\top)^\top$

1. Simulate range and azimuth measurements of the target position $\mathbf{r}_k$ with a random number generator normrnd$(0, 1)$ producing normally distributed zero-mean and unit-variance random numbers:

$$\mathbf{z}_k^p = \begin{pmatrix} z_k^r \\ z_k^\varphi \end{pmatrix} = \begin{pmatrix} \sqrt{(x_k - x_s)^2 + (y_k - y_s)^2 + (z_k - z_s)^2 - z_s^2} \\ \text{arctan}(\frac{y_k - y_s}{x_k - x_s}) \end{pmatrix} + \begin{pmatrix} \sigma_r \text{ normrnd}(0,1) \\ \sigma_\varphi \text{ normrnd}(0,1) \end{pmatrix}$$

   with σ_r = 10 m, σ_φ = 0.1° denoting the standard deviations in range and azimuth. Assume that the radars are not able to measure the elevation angle (see discussion on the whiteboard!).

2. Transform the measurements in x-y-Cartesian coordinates $z_k^r(\cos z_k^\varphi, \sin z_k^\varphi)^\top + \mathbf{r}_s$ and plot them over x-y projection of the true target trajectory! Play with sensor positions and measurement error standard deviations!

Fraunhofer FKIE

initiation: $p(\mathbf{x}_0) = \mathcal{N}(\mathbf{x}_0;\, \mathbf{x}_{0|0},\, \mathbf{P}_{0|0})$, initial ignorance: $\mathbf{P}_{0|0}$ 'large'

prediction: $\mathcal{N}(\mathbf{x}_{k-1};\, \mathbf{x}_{k-1|k-1},\, \mathbf{P}_{k-1|k-1}) \xrightarrow[\mathbf{F}_{k|k-1},\, \mathbf{D}_{k|k-1}]{\text{dynamics model}} \mathcal{N}(\mathbf{x}_k;\, \mathbf{x}_{k|k-1},\, \mathbf{P}_{k|k-1})$

$$\mathbf{x}_{k|k-1} = \mathbf{F}_{k|k-1}\mathbf{x}_{k-1|k-1}$$
$$\mathbf{P}_{k|k-1} = \mathbf{F}_{k|k-1}\mathbf{P}_{k-1|k-1}\mathbf{F}_{k|k-1}{}^{\top} + \mathbf{D}_{k|k-1}$$

filtering: $\mathcal{N}(\mathbf{x}_k;\, \mathbf{x}_{k|k-1},\, \mathbf{P}_{k|k-1}) \xrightarrow[\text{sensor model: } \mathbf{H}_k,\, \mathbf{R}_k]{\text{current measurement } \mathbf{z}_k} \mathcal{N}(\mathbf{x}_k;\, \mathbf{x}_{k|k},\, \mathbf{P}_{k|k})$

$$\begin{aligned} \mathbf{x}_{k|k} &= \mathbf{x}_{k|k-1} + \mathbf{W}_{k|k-1}\boldsymbol{\nu}_{k|k-1}, & \boldsymbol{\nu}_{k|k-1} &= \mathbf{z}_k - \mathbf{H}_k\mathbf{x}_{k|k-1} \\ \mathbf{P}_{k|k} &= \mathbf{P}_{k|k-1} - \mathbf{W}_{k|k-1}\mathbf{S}_{k|k-1}\mathbf{W}_{k|k-1}{}^{\top}, & \mathbf{S}_{k|k-1} &= \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k{}^{\top} + \mathbf{R}_k \\ \mathbf{W}_{k|k-1} &= \mathbf{P}_{k|k-1}\mathbf{H}_k{}^{\top}\mathbf{S}_{k|k-1}{}^{-1} & &\text{'KALMAN gain matrix'} \end{aligned}$$

Exercise 4.2

In your sensor simulator, chose a sensor at position $\mathbf{r}_s$ that produces x-y measurements $\mathbf{z}_k$ of the Cartesian target x-y positions $\mathbf{H}\mathbf{x}_k$ from your ground truth generator using the measurement matrix $\mathbf{H}$:

$$\mathbf{H}\mathbf{x}_k = \begin{pmatrix} 1,0,0,0,0,0 \\ 0,1,0,0,0,0 \end{pmatrix} \mathbf{x}_k$$

Calculate for each measurement the measurement error covariance matrix $\mathbf{R}_k$ based on the true target position. Program your first Kalman filter initiated by the first measurement and reasonably chosen covariance matrices $\mathbf{P}_{1|1}$. What is reasonable? Visualize nicely and compare with the truth and the measurements.

Selected exercises to be chosen from:

Retrodiction — discrete time, eventually continuous time.

Simulate missing and false plots — Realize a PDAF tracker.

Consider useful IMM parameters — two model IMM tracker.

Discretize the road — Realize a road map assisted tracker.

Create a GMTI sensor simulator — Realize a GMTI tracker.

Test the consistency of one of your trackers — NIS, NEES.