Bayesian Tracking: Basic Idea

Iterative updating of conditional probability densities!

kinematic target state x_k at time t_k, **accumulated sensor data** $\mathcal{Z}^k$

a priori knowledge: target dynamics models, sensor model, road maps

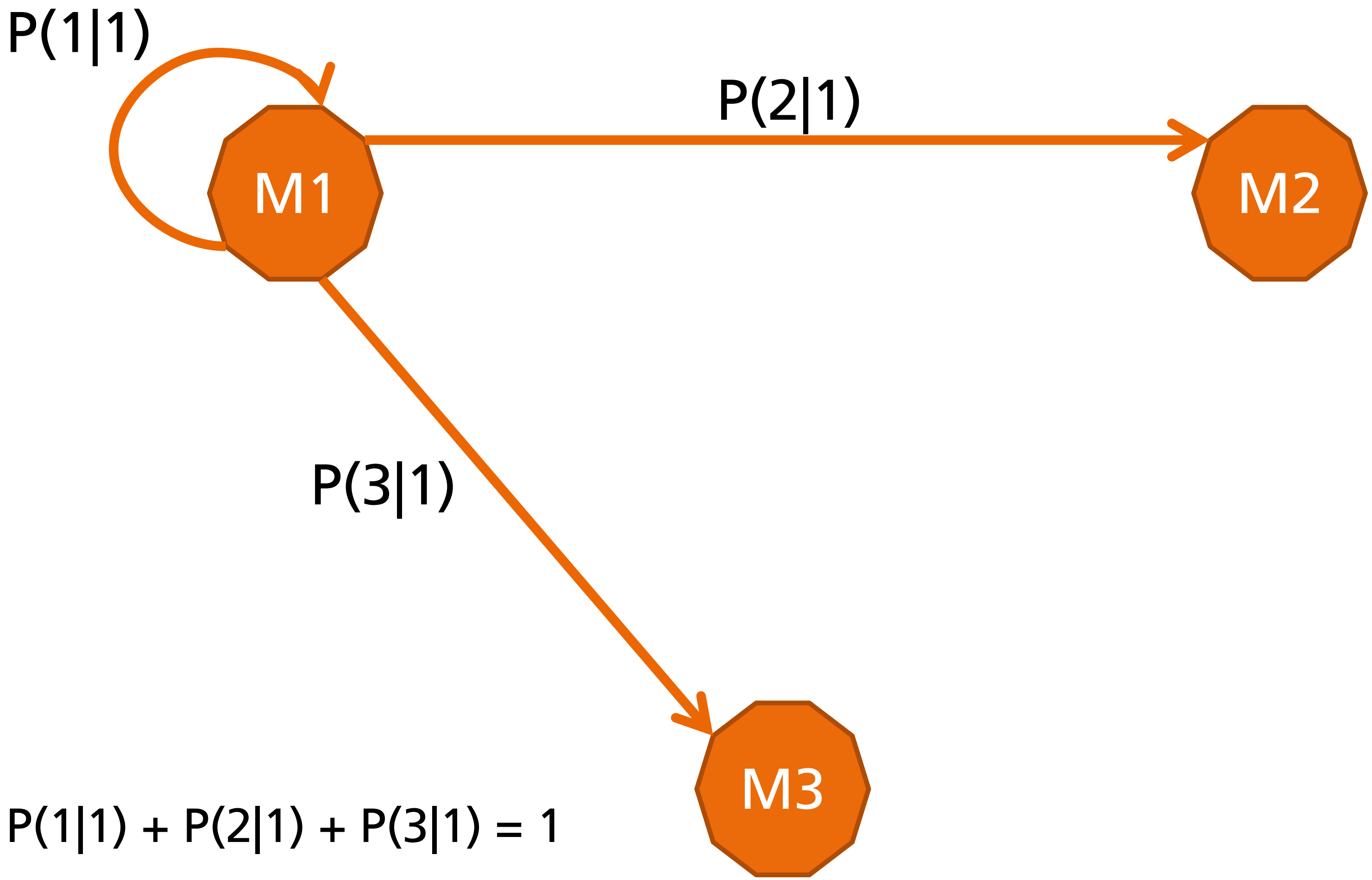
- **prediction:** $p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1}) \xrightarrow[\text{road maps}]{\text{dynamics model}} p(\mathbf{x}_k|\mathcal{Z}^{k-1})$
- **filtering:** $p(\mathrm{x}_k|\mathcal{Z}^{k-1}) \xrightarrow[\text{sensor model}]{\text{sensor data } Z_k} p(\mathrm{x}_k|\mathcal{Z}^k)$
- **retrodiction:** $p(\mathbf{x}_{l-1}|\mathcal{Z}^k) \xleftarrow[\text{dynamics model}]{\text{filtering output}} p(\mathbf{x}_l|\mathcal{Z}^k)$

– ***finite mixture:*** **inherent ambiguity (data, model, road *network*)**

– ***optimal estimators:*** **e.g. minimum mean squared error (MMSE)**

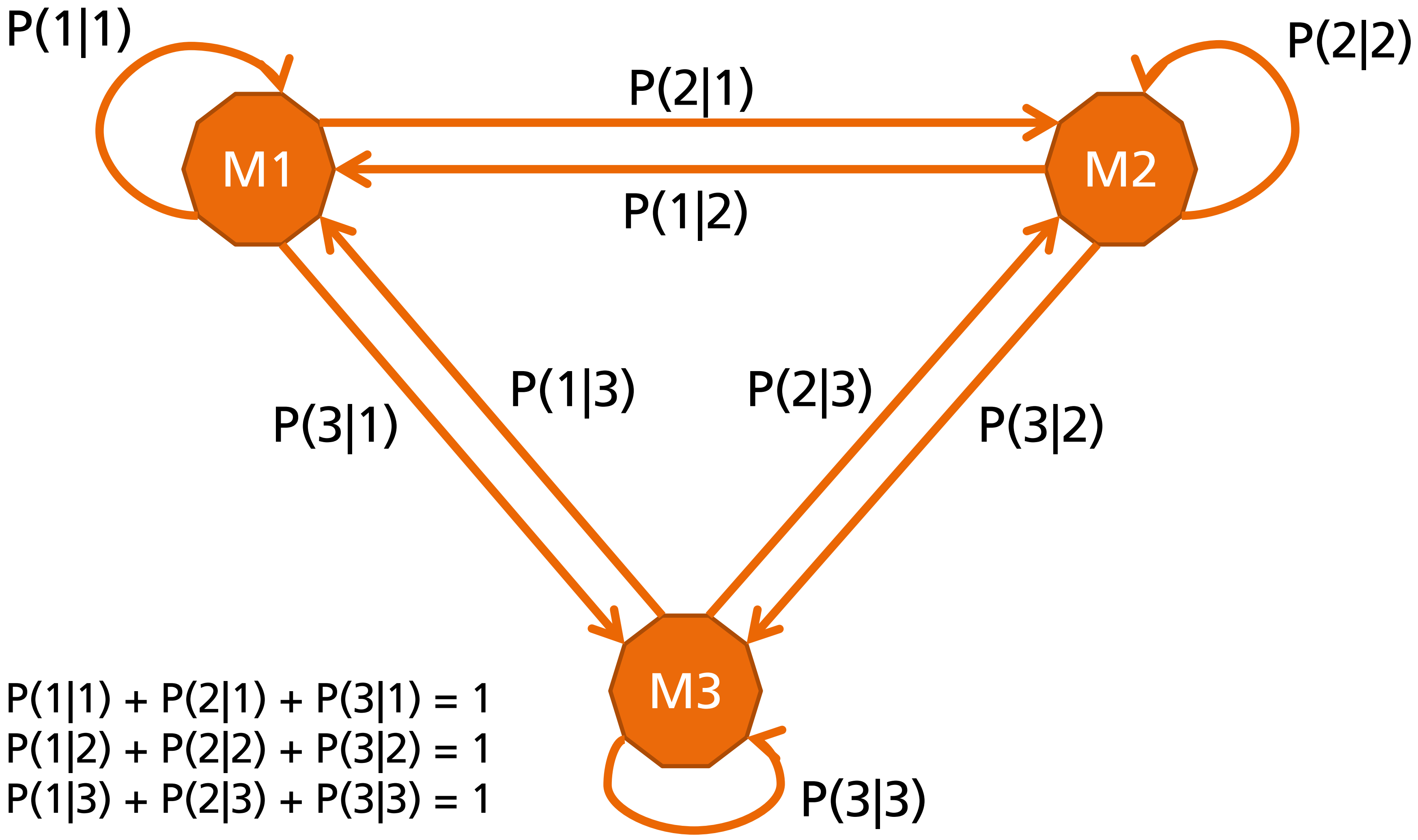
– ***initiation of pdf iteration:*** **multiple hypothesis track extraction**

Quite general: agent switching between different modes of over-all behavior



$P(1|1) + P(2|1) + P(3|1) = 1$

Quite general: agent switching between different modes of over-all behavior



$$P(1|1) + P(2|1) + P(3|1) = 1$$
$$P(1|2) + P(2|2) + P(3|2) = 1$$
$$P(1|3) + P(2|3) + P(3|3) = 1$$

multiple models: $p(x_k, j_k | x_{k-1}, j_{k-1}) = p(j_k | j_{k-1})\ \mathcal{N}\big(\mathbf{x}_k;\ \mathbf{F}^{j_k}_{k|k-1}\mathbf{x}_{k-1},\ \mathbf{D}^{j_k}_{k|k-1}\big)$

Augment the kinematic state x_k by a maneuvering class j_k.

Both quantities describe a more general 'joint state'.

While the kinematic state x_k is 'observed' by imperfect measurements z_k, the maneuvering class j_k is 'hidden', i.e. to be inferred indirectly by kinematic measurements.

In other applications called *Hidden Markov Process* HMM.

multiple models: $p(x_k, j_k | x_{k-1}, j_{k-1}) = p(j_k | j_{k-1})\ \mathcal{N}\big(\mathbf{x}_k;\ \mathbf{F}_{k|k-1}^{j_k}\mathbf{x}_{k-1},\ \mathbf{D}_{k|k-1}^{j_k}\big)$

prediction: $p(\mathbf{x}_k | \mathcal{Z}^{k-1}) = \sum_{j_k=1}^{r} \sum_{j_{k-1}=1}^{r} p(j_k | j_{k-1})\, p(j_{k-1} | \mathcal{Z}^{k-1})\ \mathcal{N}\big(\mathbf{x}_k;\ \mathbf{x}_{k|k-1}^{j_k j_{k-1}},\ \mathbf{P}_{k|k-1}^{j_k j_{k-1}}\big)$

$$\mathbf{x}_{k|k-1}^{j_k j_{k-1}} = \mathbf{F}_{k|k+1}^{j_k}\mathbf{x}_{k-1|k-1}^{j_{k-1}}, \qquad \mathbf{P}_{k|k-1}^{j_k j_{k-1}} = \mathbf{F}_{k|k+1}^{j_k}\mathbf{x}_{k-1|k-1}^{j_{k-1}}\mathbf{F}_{k|k+1}^{j_k\top} + \mathbf{D}_{k|k+1}^{j_k}$$

multiple models: $p(x_k, j_k | x_{k-1}, j_{k-1}) = p(j_k | j_{k-1}) \; \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{F}^{j_k}_{k|k-1}\mathbf{x}_{k-1},\, \mathbf{D}^{j_k}_{k|k-1}\big)$

prediction: $p(\mathbf{x}_k | \mathcal{Z}^{k-1}) = \sum_{j_k=1}^{r} \sum_{j_{k-1}=1}^{r} p(j_k | j_{k-1}) \, p(j_{k-1} | \mathcal{Z}^{k-1}) \; \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}^{j_k j_{k-1}}_{k|k-1},\, \mathbf{P}^{j_k j_{k-1}}_{k|k-1}\big)$

$$\mathbf{x}^{j_k j_{k-1}}_{k|k-1} = \mathbf{F}^{j_k}_{k|k+1}\mathbf{x}^{j_{k-1}}_{k-1|k-1}, \qquad \mathbf{P}^{j_k j_{k-1}}_{k|k-1} = \mathbf{F}^{j_k}_{k|k+1}\mathbf{x}^{j_{k-1}}_{k-1|k-1}\mathbf{F}^{j_k\top}_{k|k+1} + \mathbf{D}^{j_k}_{k|k+1}$$

mixing step: $p(\mathbf{x}_k | \mathcal{Z}^{k-1}) \approx \sum_{j_k=1}^{r} p(j_k | \mathcal{Z}^{k-1}) \; \mathcal{N}(\mathbf{x}_k;\, \mathbf{x}^{j_k}_{k|k-1},\, \mathbf{P}^{j_k}_{k|k-1})$

$$p(j_k | \mathcal{Z}^{k-1}) = \sum\nolimits_{j_{k-1}=1}^{r} p(j_k | j_{k-1}) \, p(j_{k-1} | \mathcal{Z}^{k-1})$$

$$\mathbf{x}^{j_k}_{k|k-1} = \frac{1}{p(j_k|\mathcal{Z}^{k-1})} \sum\nolimits_{j_{k-1}=1}^{r} p(j_k | j_{k-1}) \, p(j_{k-1} | \mathcal{Z}^{k-1}) \mathbf{x}^{j_k j_{k-1}}_{k|k-1}, \quad \mathbf{P}^{j_k}_{k|k-1} = \frac{1}{p(j_k|\mathcal{Z}^{k-1})} \sum\nolimits_{j_{k-1}=1}^{r} p(j_k | j_{k-1}) \, p(j_{k-1} | \mathcal{Z}^{k-1}) \big(\mathbf{P}^{j_k j_{k-1}}_{k|k-1} + (\mathbf{x}^{j_k j_{k-1}}_{k|k-1} - \mathbf{x}^{j_k}_{k|k-1})(\cdots)^\top\big)$$

multiple models: $p(x_k, j_k | x_{k-1}, j_{k-1}) = p(j_k | j_{k-1}) \; \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{F}^{j_k}_{k|k-1}\mathbf{x}_{k-1},\, \mathbf{D}^{j_k}_{k|k-1}\big)$

prediction: $$p(\mathbf{x}_k | \mathcal{Z}^{k-1}) = \sum_{j_k=1}^{r} \sum_{j_{k-1}=1}^{r} p(j_k | j_{k-1})\, p(j_{k-1} | \mathcal{Z}^{k-1}) \; \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}^{j_k j_{k-1}}_{k|k-1},\, \mathbf{P}^{j_k j_{k-1}}_{k|k-1}\big)$$

$$\mathbf{x}^{j_k j_{k-1}}_{k|k-1} = \mathbf{F}^{j_k}_{k|k+1}\mathbf{x}^{j_{k-1}}_{k-1|k-1}, \qquad \mathbf{P}^{j_k j_{k-1}}_{k|k-1} = \mathbf{F}^{j_k}_{k|k+1}\mathbf{x}^{j_{k-1}}_{k-1|k-1}\mathbf{F}^{j_k \top}_{k|k+1} + \mathbf{D}^{j_k}_{k|k+1}$$

mixing step: $$p(\mathbf{x}_k | \mathcal{Z}^{k-1}) \approx \sum_{j_k=1}^{r} p(j_k | \mathcal{Z}^{k-1}) \; \mathcal{N}(\mathbf{x}_k;\, \mathbf{x}^{j_k}_{k|k-1},\, \mathbf{P}^{j_k}_{k|k-1})$$

$$p(j_k | \mathcal{Z}^{k-1}) = \sum\nolimits_{j_{k-1}=1}^{r} p(j_k | j_{k-1})\, p(j_{k-1} | \mathcal{Z}^{k-1})$$

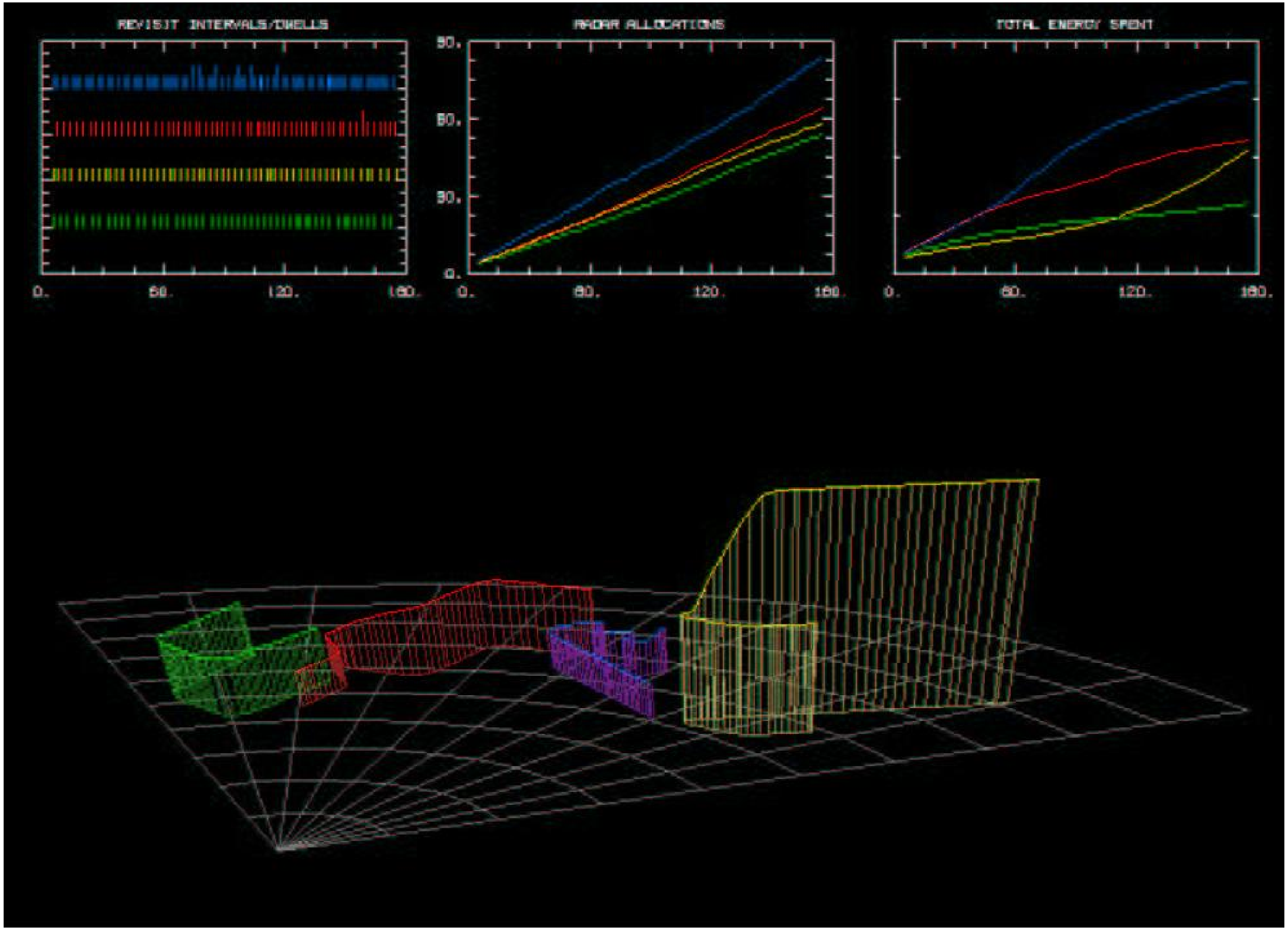
$$\mathbf{x}^{j_k}_{k|k-1} = \frac{1}{p(j_k|\mathcal{Z}^{k-1})} \sum\nolimits_{j_{k-1}=1}^{r} p(j_k | j_{k-1})\, p(j_{k-1} | \mathcal{Z}^{k-1}) \mathbf{x}^{j_k j_{k-1}}_{k|k-1}, \qquad \mathbf{P}^{j_k}_{k|k-1} = \frac{1}{p(j_k|\mathcal{Z}^{k-1})} \sum\nolimits_{j_{k-1}=1}^{r} p(j_k | j_{k-1})\, p(j_{k-1} | \mathcal{Z}^{k-1}) \Big(\mathbf{P}^{j_k j_{k-1}}_{k|k-1} + (\mathbf{x}^{j_k j_{k-1}}_{k|k-1} - \mathbf{x}^{j_k}_{k|k-1})(\cdots)^\top\Big)$$

filtering: $$p(\mathbf{x}_k | \mathcal{Z}^{k}) = \sum_{j_k=1}^{r} p(j_k | \mathcal{Z}^{k}) \; \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}^{j_k}_{k|k},\, \mathbf{P}^{j_k}_{k|k}\big)$$

with: $$p(j_k | \mathcal{Z}^k) = \frac{\mathcal{N}\Big(\mathbf{z}_k;\, \mathbf{H}_k \mathbf{x}^{j_k}_{k|k-1},\, \mathbf{H}_k \mathbf{P}^{j_k}_{k|k-1} \mathbf{H}_k + \mathbf{R}_k\Big)}{\sum\nolimits_{j'_k=1}^{r} p(j'_k | \mathcal{Z}^{k-1}) \; \mathcal{N}\Big(\mathbf{z}_k;\, \mathbf{H}_k \mathbf{x}^{j'_k}_{k|k-1},\, \mathbf{H}_k \mathbf{P}^{j'_k}_{k|k-1} \mathbf{H}_k + \mathbf{R}_k\Big)}$$ (mixture coefficients)

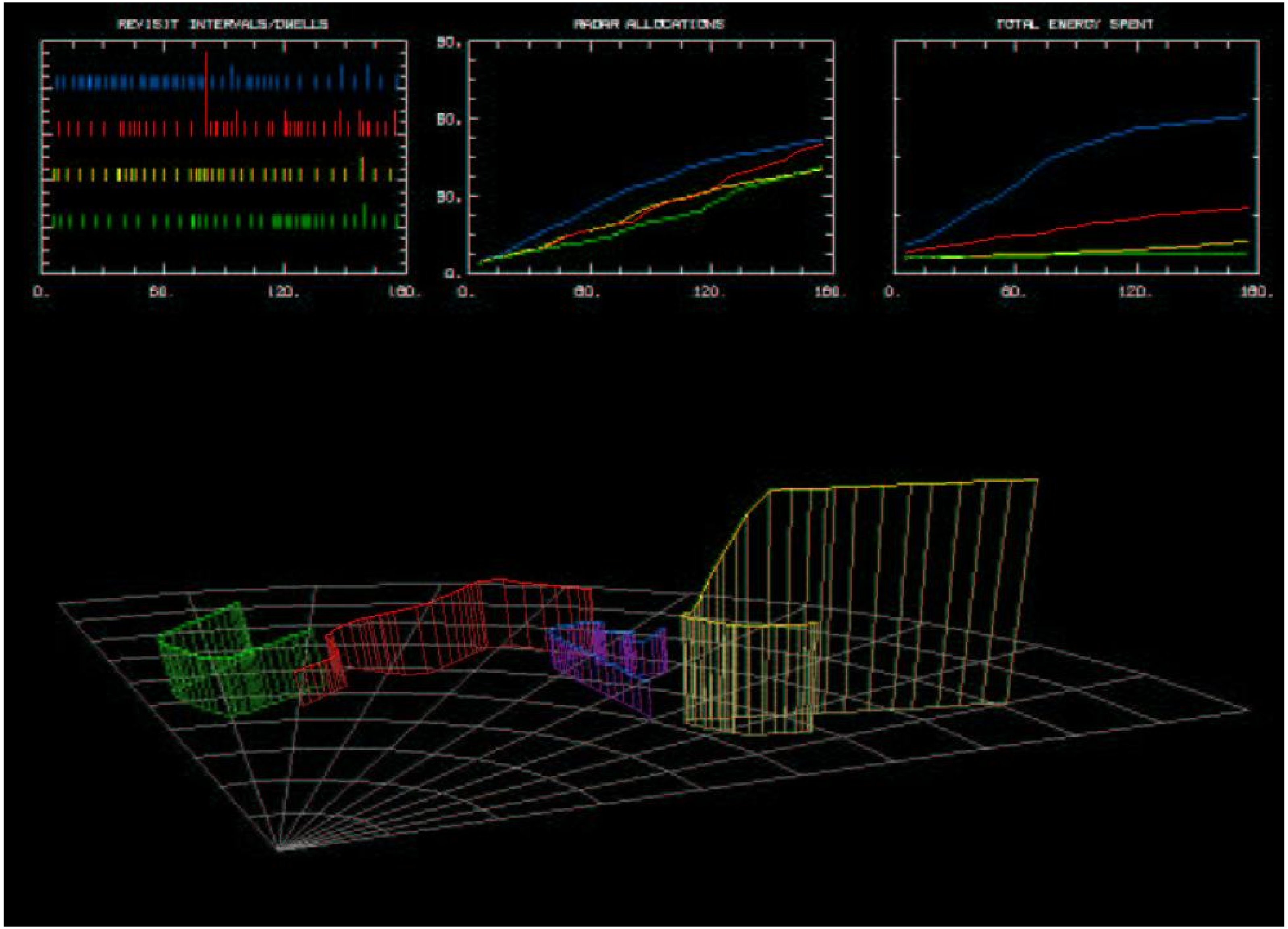
$$\mathbf{x}^{j_k}_{k|k} = \mathbf{x}^{j_k}_{k|k-1} + \mathbf{W}^{j_k}_{k|k}(\mathbf{z}_k - \mathbf{H}_k \mathbf{x}^{j_k}_{k|k-1}), \quad \mathbf{P}^{j_k}_{k|k} = \mathbf{P}^{j_k}_{k|k-1} - \mathbf{W}^{j_k}_{k|k-1} \mathbf{S}^{j_k}_{k|k} \mathbf{W}^{j_k}_{k|k-1}, \quad \mathbf{W}^{j_k}_{k|k} = \mathbf{P}^{j_k}_{k|k-1} \mathbf{H}^\top_k \mathbf{S}^{j_k^{-1}}_{k|k} \quad \mathbf{S}^{j_k}_{k|k} = \mathbf{H}_k \mathbf{P}^{j_k}_{k|k-1} \mathbf{H}_k + \mathbf{R}_k$$

Phased-Array Tracking: Standard Sensor Management



Fraunhofer FKIE

Phased-Array Tracking: Adaptive Sensor Management



IMM Models: Retrodiction

$$p(\mathbf{x}_l|\mathcal{Z}^k) = \sum_{i_{l+1}, i_l} \int d\mathbf{x}_{l+1}\, p(\mathbf{x}_{l+1}, \mathbf{x}_l, i_{l+1}, i_l|\mathcal{Z}^k)$$ (marginal density!)

IMM Models: Retrodiction

$$p(\mathbf{x}_l|\mathcal{Z}^k) = \sum_{i_{l+1}, i_l} \int d\mathbf{x}_{l+1}\, p(\mathbf{x}_{l+1}, \mathbf{x}_l, i_{l+1}, i_l|\mathcal{Z}^k) \qquad \text{(marginal density!)}$$

$$= \sum_{i_{l+1}, i_l} \int d\mathbf{x}_{l+1}\, p(\mathbf{x}_l, i_l|\mathbf{x}_{l+1}, i_{l+1}\mathcal{Z}^k)\, \underbrace{p(\mathbf{x}_{l+1}, i_{l+1}|\mathcal{Z}^k)}_{\text{retrodiction for } t_{l+1}}$$

for time $l+1$ assume: $p(\mathbf{x}_{l+1}|\mathcal{Z}^k) = \sum_{i_{l+1}} p(\mathbf{x}_{l+1}, i_{l+1}|\mathcal{Z}^k) = \sum_{i_{l+1}} \boldsymbol{\mu}^k_{i_{l+1}}\, \mathcal{N}\big(\mathbf{x}_{l+1};\, \mathbf{x}^k_{i_{l+1}},\, \mathbf{P}^k_{i_{l+1}}\big)$

IMM Models: Retrodiction

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$$p(\mathbf{x}_l, i_l|\mathbf{x}_{l+1}, i_{l+1}, \boxed{\mathcal{Z}^k}) = p(\mathbf{x}_l, i_l|\mathbf{x}_{l+1}, i_{l+1}, \boxed{\mathcal{Z}^l}) = \frac{p(\mathbf{x}_{l+1}, i_{l+1}|\mathbf{x}_l, i_l)\, p(\mathbf{x}_l, i_l|\mathcal{Z}^l)}{\sum_{i_l} \int \mathbf{x}_l\, \underbrace{p(\mathbf{x}_{l+1}, i_{l+1}|\mathbf{x}_l, i_l)}_{\text{IMM model}}\, \underbrace{p(\mathbf{x}_l, i_l|\mathcal{Z}^l)}_{\text{filtering } t_l}}$$

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$$p(\mathbf{x}_l, i_l|\mathbf{x}_{l+1}, i_{l+1}, \boxed{\mathcal{Z}^k}) = p(\mathbf{x}_l, i_l|\mathbf{x}_{l+1}, i_{l+1}, \boxed{\mathcal{Z}^l}) = \frac{p(\mathbf{x}_{l+1}, i_{l+1}|\mathbf{x}_l, i_l)\; p(\mathbf{x}_l, i_l|\mathcal{Z}^l)}{\sum_{i_l} \int \mathbf{x}_l\; \underbrace{p(\mathbf{x}_{l+1}, i_{l+1}|\mathbf{x}_l, i_l)}_{\text{IMM model}}\; \underbrace{p(\mathbf{x}_l, i_l|\mathcal{Z}^l)}_{\text{filtering } t_l}}$$

$$= \mathbf{c}_{i_{l+1}i_l}(\mathbf{x}_{l+1})\; \mathcal{N}\Big(\mathbf{x}_l;\; \mathbf{x}_{i_l} + \mathbf{W}_{i_{l+1}i_l}\big(\mathbf{x}_{l+1} - \mathbf{x}_{i_{l+1}i_l}\big),\; \mathbf{P}_{i_l} - \mathbf{W}_{i_{l+1}i_l}\mathbf{P}_{i_{l+1}i_l}\mathbf{W}_{i_{l+1}i_l}{}^\top\Big) \qquad \textbf{product formula!}$$

with: $\mathbf{c}_{i_{l+1}i_l}(\mathbf{x}_{l+1}) = \dfrac{\boldsymbol{\mu}_{i_{l+1}i_l}\; \mathcal{N}\big(\mathbf{x}_{l+1};\; \mathbf{x}_{i_{l+1}i_l},\; \mathbf{P}_{i_{l+1}i_l}\big)}{\sum_{i_l} \boldsymbol{\mu}_{i_{l+1}i_l}\; \mathcal{N}\big(\mathbf{x}_{l+1};\; \mathbf{x}_{i_{l+1}i_l},\; \mathbf{P}_{i_{l+1}i_l}\big)}$ $\qquad \mathbf{W}_{i_{l+1}i_l} = \mathbf{P}_{i_l}\mathbf{F}^\top_{i_{l+1}}(\mathbf{F}_{i_{l+1}}\mathbf{P}_{i_l}\mathbf{F}^\top_{i_{l+1}} + \mathbf{D}_{i_{l+1}})^{-1}$

$$\approx \mathbf{c}_{i_{l+1}i_l}(\mathbf{x}^k_{i_{l+1}})$$

Fraunhofer FKIE

$$p(\mathbf{x}_l|\mathcal{Z}^k) = \sum_{i_{l+1}, i_l} \int d\mathbf{x}_{l+1} \underbrace{p(\mathbf{x}_l, i_l|\mathbf{x}_{l+1}, i_{l+1}, \mathcal{Z}^k)}_{\text{calculated!}} \underbrace{p(\mathbf{x}_{l+1}, i_{l+1}|\mathcal{Z}^k)}_{\text{retrodiction in } t_{l+1}}$$

insert, product formula!

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insert, product formula!

$$= \sum_{i_{l+1}, i_l} \boldsymbol{\mu}^k_{i_{l+1} i_l} \, \mathcal{N}\Big(\mathbf{x}_l;\, \mathbf{x}^k_{i_{l+1} i_l},\, \mathbf{P}^k_{i_{l+1} i_l}\Big)$$

exponential growth of dynamics histories $i_{i_{l+1} i_l \ldots}$!

$$p(\mathbf{x}_l|\mathcal{Z}^k) = \sum_{i_{l+1}, i_l} \int d\mathbf{x}_{l+1} \underbrace{p(\mathbf{x}_l, i_l|\mathbf{x}_{l+1}, i_{l+1}, \mathcal{Z}^k)}_{\text{calculated!}} \underbrace{p(\mathbf{x}_{l+1}, i_{l+1}|\mathcal{Z}^k)}_{\text{retrodiction in } t_{l+1}}$$

insert, product formula!

$$= \sum_{i_{l+1}, i_l} \boldsymbol{\mu}^k_{i_{l+1} i_l} \, \mathcal{N}\Big(\mathbf{x}_l;\, \mathbf{x}^k_{i_{l+1} i_l},\, \mathbf{P}^k_{i_{l+1} i_l}\Big)$$

exponential growth of dynamics histories $i_{i_{l+1} i_l \dots}$!

$$= \sum_{i_l} \underbrace{\sum_{i_{l+1}} \boldsymbol{\mu}^k_{i_{l+1} i_l} \, \mathcal{N}\Big(\mathbf{x}_l;\, \mathbf{x}^k_{i_{l+1} i_l},\, \mathbf{P}^k_{i_{l+1} i_l}\Big)}_{\text{approximation: moment matching!}}$$

finally: $p(\mathbf{x}_l|\mathcal{Z}^k) \approx \sum_{i_l} \boldsymbol{\mu}^k_{i_l} \, \mathcal{N}\Big(\mathbf{x}_l;\, \mathbf{x}^k_{i_l},\, \mathbf{P}^k_{i_l}\Big)$

generalize: model histories of *variable* length!

IMM Modeling: Suboptimal Realization

- **Conventional KALMAN filtering**

  Only *one* component: worst-case assumtion

- **standard IMM filter (as discussed!)**

  Approximate after prediction, *before* update by r components! Effort: $\sim r$ KALMAN filter

- **GPB: Generalized Pseudo-BAYESian**

  Approximate *after* measurement processing by r components! Effort: $\sim r^2$ KALMAN filter

- **IMM-MHT filter (nearly optimal)**

  Accept longer dynamics histories $\rightarrow$ *variable* number of components!

Extendable to ambiguity with respect to sensor models!

Improved Approximation: Simple Approach

Consider ***dynamics histories*** **of length** κ**:** $\mathbf{i}_k = (i_k, i_{k-1}, \ldots, i_{k-\kappa+1})$

Improved Approximation: Simple Approach

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als Filterung:

$$p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1}) = \underbrace{\sum_{\mathbf{i}_{k-1}}}_{\kappa \text{ Summen}} \boldsymbol{\mu}_{\mathbf{i}_{k-1}} \, \mathcal{N}\big(\mathbf{x}_{k-1};\, \mathbf{x}_{\mathbf{i}_{k-1}},\, \mathbf{P}_{\mathbf{i}_{k-1}}\big)$$

Improved Approximation: Simple Approach

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prediction: $$p(\mathbf{x}_k|\mathcal{Z}^{k-1}) = \sum_{i_k, \mathbf{i}_{k-1}} \int d\mathbf{x}_{k-1}\, p(\mathbf{x}_k, i_k, \mathbf{x}_{k-1}, \mathbf{i}_{k-1}|\mathcal{Z}^{k-1})$$

Improved Approximation: Simple Approach

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Prďż˝diktion:

$$p(\mathbf{x}_k|\mathcal{Z}^{k-1}) = \sum_{i_k, \mathbf{i}_{k-1}} \int d\mathbf{x}_{k-1}\, p(\mathbf{x}_k, i_k, \mathbf{x}_{k-1}, \mathbf{i}_{k-1}|\mathcal{Z}^{k-1})$$

$$= \underbrace{\sum_{i_k, \dots, i_{k-\kappa}}}_{\kappa+1 \text{ Summen}} \int d\mathbf{x}_{k-1}\, p(\mathbf{x}_k, i_k|\mathbf{x}_{k-1}, i_{k-1})\, p(\mathbf{x}_{k-1}, i_{k-1}, \dots, i_{k-\kappa}|\mathcal{Z}^{k-1})$$

Improved Approximation: Simple Approach

Consider *dynamics histories* of length κ: $\quad \mathbf{i}_k = (i_k, i_{k-1}, \ldots, i_{k-\kappa+1})$

as filtering:

$$p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1}) = \underbrace{\sum_{\mathbf{i}_{k-1}}}_{\kappa \text{ sums}} \boldsymbol{\mu}_{\mathbf{i}_{k-1}} \mathcal{N}\big(\mathbf{x}_{k-1};\, \mathbf{x}_{\mathbf{i}_{k-1}},\, \mathbf{P}_{\mathbf{i}_{k-1}}\big)$$

prediction:

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$$= \sum_{i_k, \ldots, i_{k-\kappa}} \boldsymbol{\mu}_{i_k, \ldots, i_{k-\kappa}} \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{i_k, \ldots, i_{k-\kappa}},\, \mathbf{P}_{i_k, \ldots, i_{k-\kappa}}\big) = \sum_{\mathbf{i}_k} \underbrace{\sum_{i_{k-\kappa}} \boldsymbol{\mu}_{\mathbf{i}_k, i_{k-\kappa}} \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{\mathbf{i}_k, i_{k-\kappa}},\, \mathbf{P}_{\mathbf{i}_k, i_{k-\kappa}}\big)}_{\text{second order approximation!}}$$

Fraunhofer FKIE

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Consider *dynamics histories* of length κ: $\mathbf{i}_k = (i_k, i_{k-1}, \ldots, i_{k-\kappa+1})$

as filtering:

$$p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1}) = \underbrace{\sum_{\mathbf{i}_{k-1}}}_{\kappa \text{ sums}} \boldsymbol{\mu}_{\mathbf{i}_{k-1}} \mathcal{N}\big(\mathbf{x}_{k-1};\, \mathbf{x}_{\mathbf{i}_{k-1}},\, \mathbf{P}_{\mathbf{i}_{k-1}}\big)$$

prediction:

$$p(\mathbf{x}_k|\mathcal{Z}^{k-1}) = \sum_{i_k, \mathbf{i}_{k-1}} \int d\mathbf{x}_{k-1}\, p(\mathbf{x}_k, i_k, \mathbf{x}_{k-1}, \mathbf{i}_{k-1}|\mathcal{Z}^{k-1})$$

$$= \underbrace{\sum_{i_k,\ldots,i_{k-\kappa}}}_{\kappa+1 \text{ sums}} \int d\mathbf{x}_{k-1}\, p(\mathbf{x}_k, i_k|\mathbf{x}_{k-1}, i_{k-1})\, p(\mathbf{x}_{k-1}, i_{k-1}, \ldots, i_{k-\kappa}|\mathcal{Z}^{k-1})$$

$$= \sum_{i_k,\ldots,i_{k-\kappa}} \boldsymbol{\mu}_{i_k,\ldots,i_{k-\kappa}} \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{i_k,\ldots,i_{k-\kappa}},\, \mathbf{P}_{i_k,\ldots,i_{k-\kappa}}\big) = \sum_{\mathbf{i}_k} \underbrace{\sum_{i_{k-\kappa}} \boldsymbol{\mu}_{\mathbf{i}_k, i_{k-\kappa}} \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{\mathbf{i}_k, i_{k-\kappa}},\, \mathbf{P}_{\mathbf{i}_k, i_{k-\kappa}}\big)}_{\text{second order approximation!}}$$

finally: $p(\mathbf{x}_k|\mathcal{Z}^{k-1}) \approx \sum_{\mathbf{i}_k} \boldsymbol{\mu}^*_{\mathbf{i}_k} \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}^*_{\mathbf{i}_k},\, \mathbf{P}^*_{\mathbf{i}_{k-1}}\big)$ (for retrodiction analogous!)

Design of IMM Modelling

- ***number r of models:*** **relevant only for standard IMM**
- ***decisive:*** **sufficiently many Gaußian picture components**
- ***irrelevant:*** **by r or length of dynamics khistories n_H**
- ***recommendation:*** **worst/best case, histories ($r = 2$, $n_H = 3$)**
- ***benefit:*** **interpretable, close-to-reality dynamics parameters**

Demonstration

integration of available context information

- **GMTI radar: Doppler blindness (MDV)**
- **digital road-map information (GIS)**

Tracking Application: Ground Picture Production

GMTI Radar: Ground Moving Target Indicator

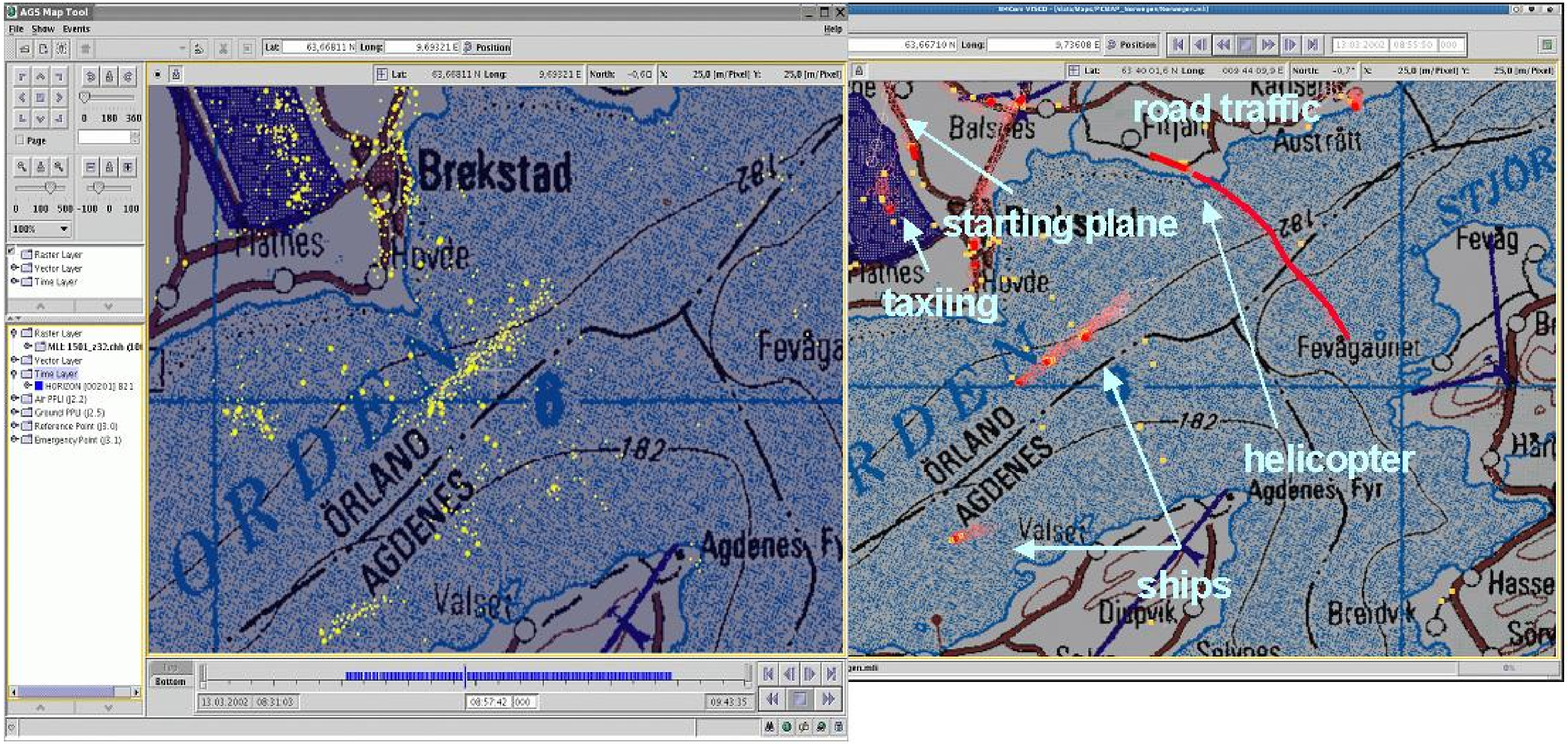
wide area, all-weather, day/night, real-time surveillance of a dynamically evolving ground or near-to-ground situation

GMTI Tracking: Some Characteristic Aspects

backbone of a ground picture: moving target tracks

- airborne, dislocated, mobile sensor platforms
- vehicles, ships, 'low-flyers', radars, convoys
- occlusions: Doppler-blindness, topography
- road maps, terrain information, tactical rules
- dense target / dense clutter situations: MHT

Examples of GMTI Tracks (live exercise)



Fraunhofer
FKIE

Cumulative Detection by N Sensors

***cumulative* detection probability** $$P_D^{\text{kum}}(N) = 1 - \prod_{n=1}^{N} (1 - P_D^n)$$

example: Doppler blindness in case of GMTI radar

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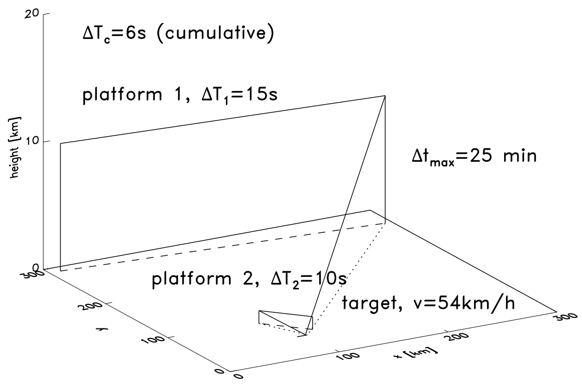

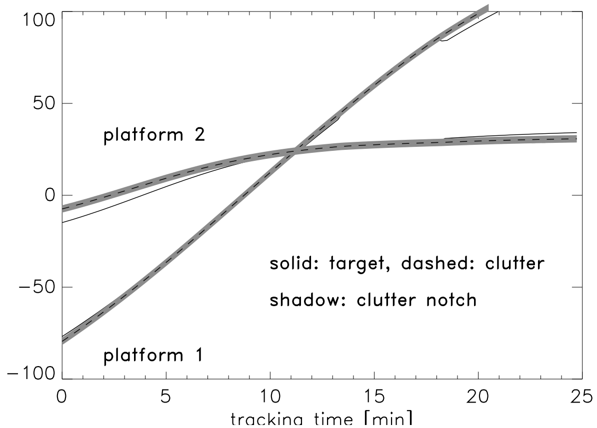

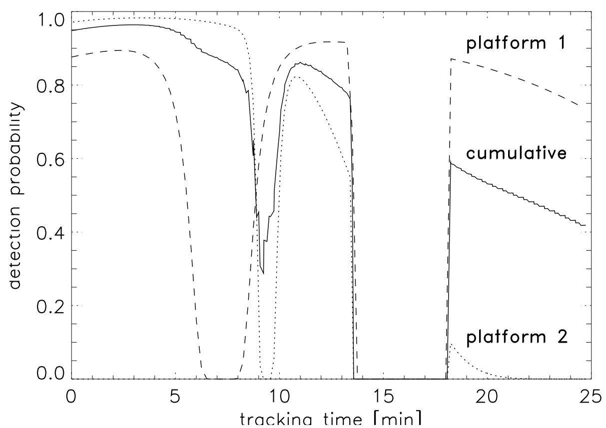

mean cumulative revisit interval:

$$1/\Delta T_c = \sum_{n=1}^{N} 1/\Delta T_n$$

mean cumulative P_D relative to ΔT_c:

$$P_D^c = 1 - \prod_{n=1}^{N} (1 - P_D^n)^{\Delta T_c/\Delta T_n}$$

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GMTI Radar: Ground Moving Target Indicator Radar

low-DOPPLER targets can be masked by the GMTI clutter notch

- *fading:* series of missing plots (target/sensor geometry)
- *stopping targets:* indistinguishable from ground clutter
- *mdv:* minimum detectable velocity (sensor parameter)

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a simple GMTI detection model: qualitative discussion

- detection depends on target kinematics & target/sensor geometry
- detection probability $P_D(\mathrm{x}_k)$ is small if $\dot{r}_k - \dot{r}_{\mathsf{mlc}}(\mathrm{x}_k) < \textit{mdv}$
- there exists a narrow transition region between these domains
- the state-dependent $P_D(\mathrm{x}_k)$ is part of the likelihood function

sensor performance: *quantitative* model

- **basis:** $\mathsf{snir} = \mathsf{snir}(r_k, \varphi_k, \dot{r}_k)$, **Signal-to-Noise+Interference Ratio**

$$\mathsf{snir} = \mathsf{snir}_0 \underbrace{\left(\frac{\bar{\sigma}_k}{\sigma_0}\right)}_{\text{rcs}} \underbrace{\left(\frac{r_k}{r_0}\right)^{-4}}_{\text{propagation}} \underbrace{D(\varphi_k)}_{\text{directivity}} \underbrace{\left[1 - \mathsf{e}^{-\log 2 \left(\frac{n_c(r_k, \varphi_k, \dot{r}_k)}{v_m}\right)^2}\right]}_{\text{clutter notch } (<\frac{1}{2} \text{ for } |n_c|<v_m)}$$

- **quadrature detector with given P_{FA}, rcs fluctuations: SWERLING I**

$$P_D(r_k, \varphi_k, \dot{r}_k) = P_{\mathsf{FA}}^{\frac{1}{1+\mathsf{snir}}} \approx P_d \left(1 - \mathsf{e}^{-\log 2 \left(\frac{n_c(r_k, \varphi_k, \dot{r}_k)}{v_m}\right)^2}\right)$$

- **as usual: residual clutter; bias free, GAUSSian errors (monopulse)**

$$\sigma_{r,\varphi,\dot{r}}(r_k, \varphi_k, \dot{r}_k) = \Sigma_{r,\varphi,\dot{r}} / \sqrt{\mathsf{snir}(r_k, \varphi_k, \dot{r}_k)}$$

filtering: $p(\mathbf{x}_k|\mathcal{Z}^{k-1}) \xrightarrow[\text{sensor modell}]{\text{sensor data } Z_k} p(\mathbf{x}_k|\mathcal{Z}^k)$

$$p(\mathbf{x}_k|\mathcal{Z}^k) = \frac{p(Z_k, n_k|\mathbf{x}_k)\, p(\mathbf{x}_k|\mathcal{Z}^{k-1})}{\int d\mathbf{x}_k\, \underbrace{p(Z_k, n_k|\mathbf{x}_k)}_{\text{likelihood}}\, \underbrace{p(\mathbf{x}_k|\mathcal{Z}^{k-1})}_{\text{prediction}}} \qquad \textbf{(BAYES)}$$

likelihood (depending on sensor data, modeling parameters):

$$p(Z_k, n_k|\mathbf{x}_k) \propto \pi_0^0 + \sum_{n=0}^{n_k} \sum_{i=0}^{1} \pi_n^i\, \mathcal{N}\Big(\mathbf{z}_k^{ni};\, \mathbf{H}_{k|k-1}^{ni}\mathbf{x}_k,\, \mathbf{R}_{k|k-1}^{ni}\Big)_{n\neq 0, i\neq 0}$$

(after some calculations and mild approximations; π_n^i constant coefficients)

essential: exploit state dependency of $P_d(\mathbf{x}_k)$!

Clutter Notch: A Priori Information!

- *current* position (Sensor-to-target-geometry)
- sensor specific *width* (STAP $\rightarrow$ MDV)
- detection process: *generic* model

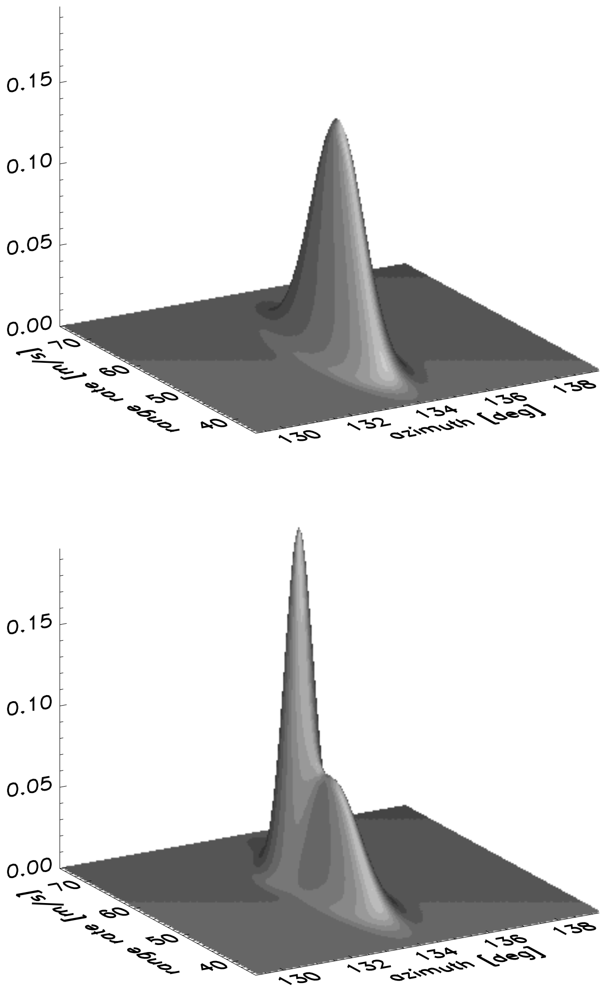
GMTI model $\Rightarrow$ mixture densities:

- well-understood formalism directly applicable
- class of GAUSS*ian mixtures* remains invariant
- pdfs characterized by a *set of parameters*
- growing memory: standard-type approximations

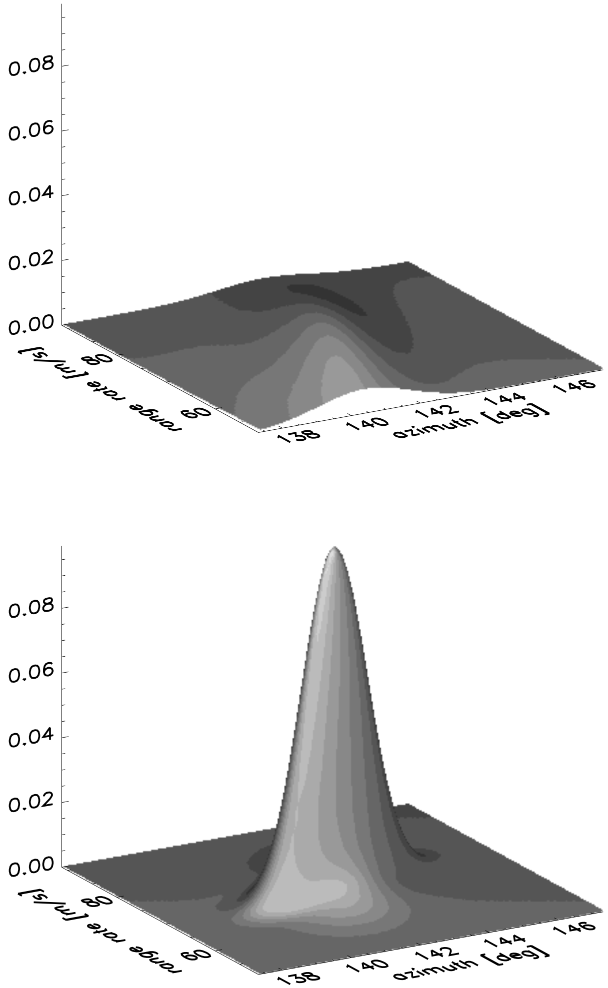
***model inherent:* reason for missing detections**

$\Rightarrow$ An adequate treatment becomes possible!

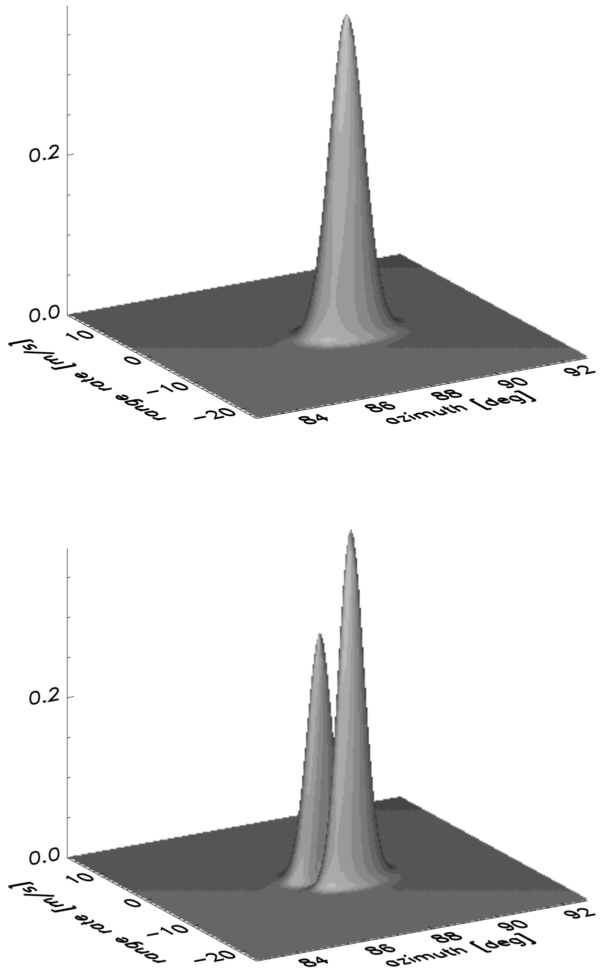
PDFs with / without exploiting the GMTI sensor model



Missing detection occurred near the clutter notch



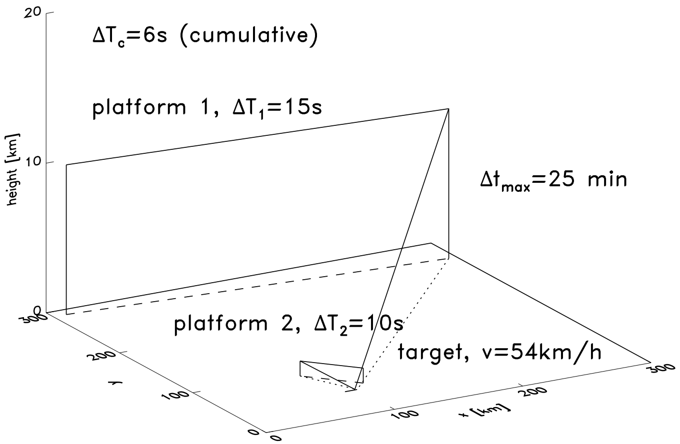
Several missing detections in the clutter notch

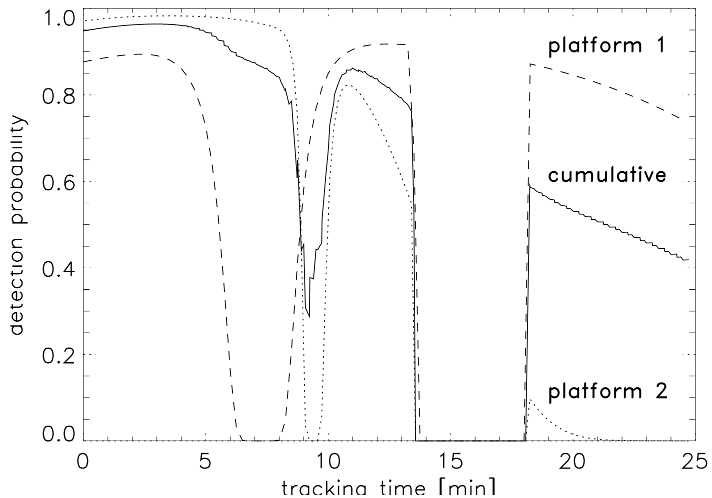


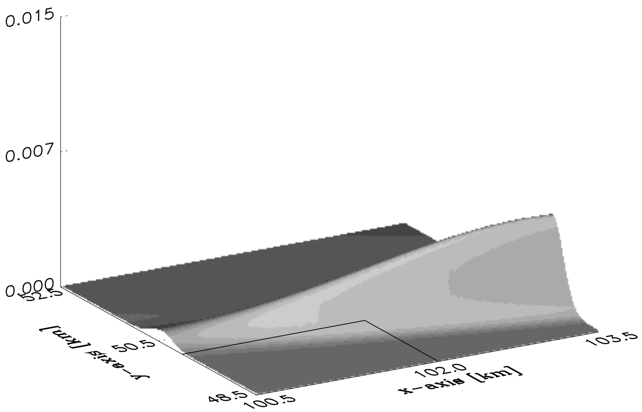
Detection occurred near the clutter notch

Fraunhofer FKIE

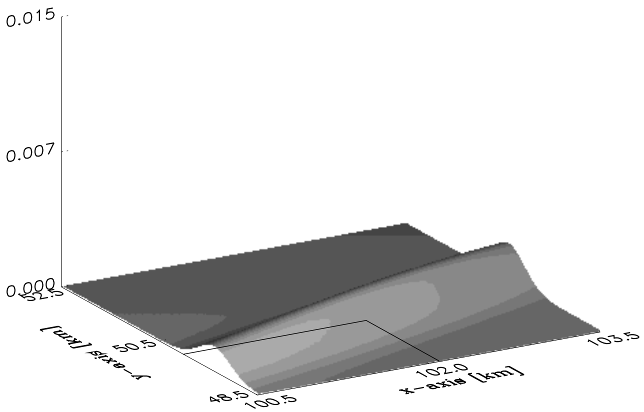
early detection of stopping targets



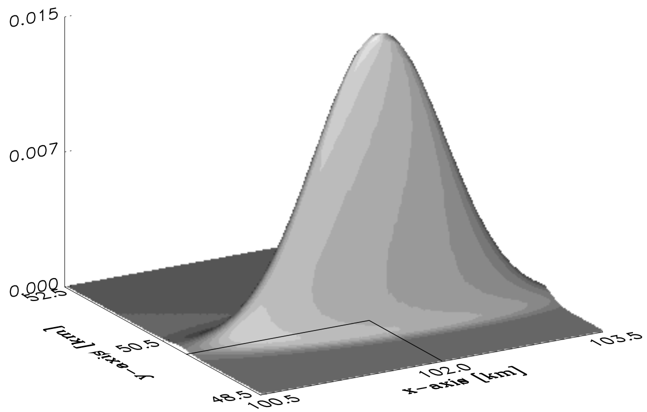




neg. output sensor 1



neg. output sensor 2



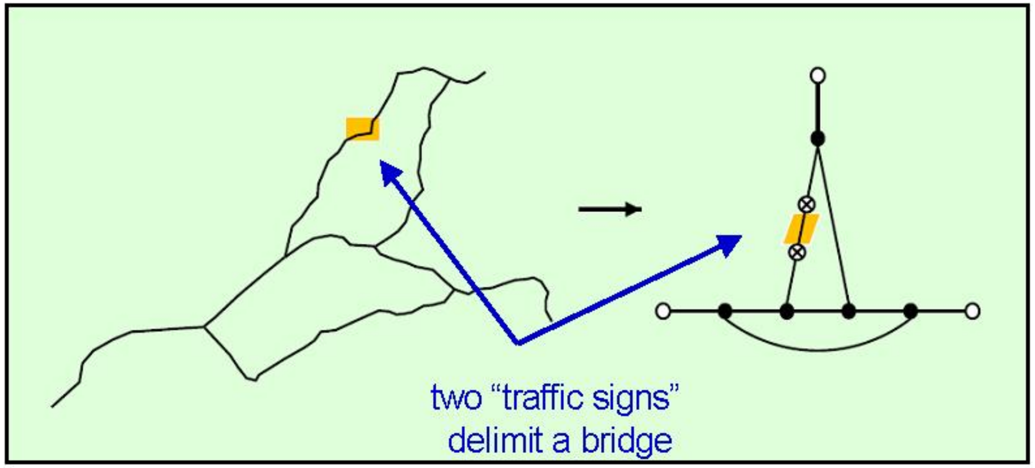
fusion: sensor 1+2

A *‘negative’ sensor output* can also provide information on the kinematical state vector of a target.

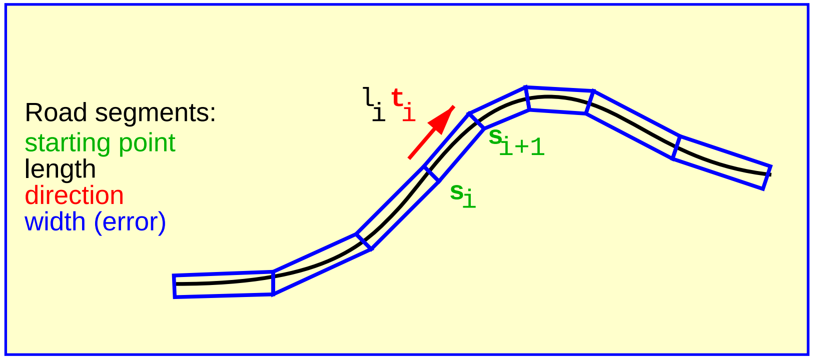
- ***fictitious plot*: function of position / radial speed**
- ***mdv*: appears as a fictitious measurement error**
- ***fusion*: exploit differing target/sensor geometries**

$$\ell(\mathrm{x}_k; Z_k) = (1 - P_D(\mathrm{x}_k)\rho_F + P_D(\mathrm{x}_k) \sum_{j=1}^{m_k} \mathcal{N}\Big(\mathbf{z}_k^j;\, \mathbf{H}\mathbf{x}_k,\, \mathbf{R}_k^j\Big)$$

a simple model for road networks



nodes: sources, crossings, 'traffic signs'
edges: representation of road segments



Fraunhofer FKIE

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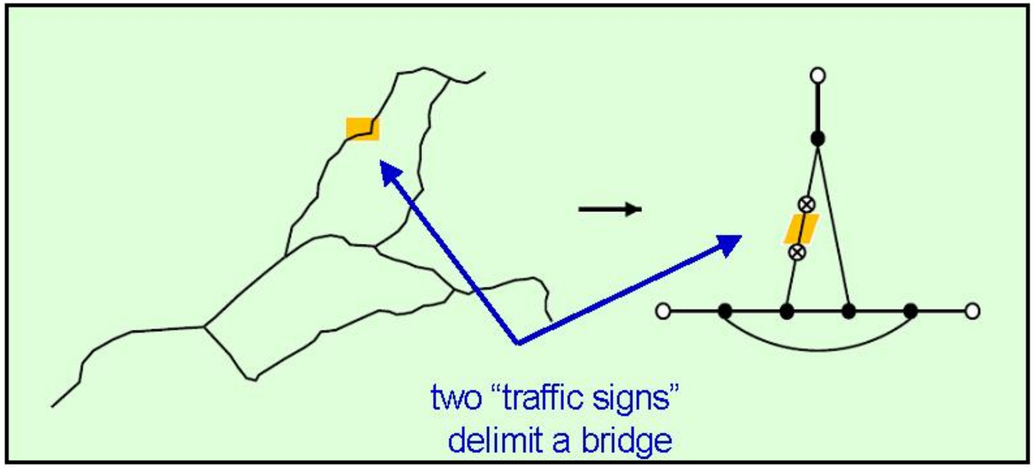

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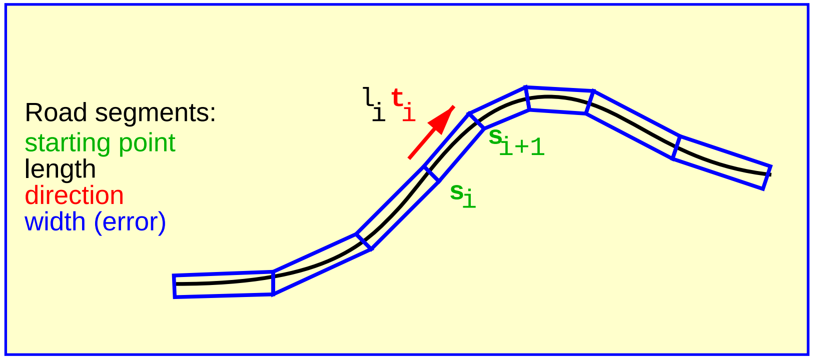

approximation of a road segment: $$\mathcal{R} : l \in [l_1, l_{n_r}) \mapsto \mathcal{R}(l) = \sum_{m=1}^{n_r} \Big[\mathbf{s}_m + (l - l_m)\mathbf{t}_m\Big]\chi_m(l)$$

arc length l, node vector $\mathbf{s}_m = \mathcal{R}(l_m)$, tangential vector $\mathbf{t}_m$, # of nodes n_r

accuracy of $\mathbf{s}_m$: covariance matrix $\mathbf{R}_m$, $\chi_m(l) = \begin{cases} 1 & \text{for } l \in [l_m, l_{m+1}) \\ 0 & \text{else} \end{cases}$

in general: $||\mathbf{s}_m - \mathbf{s}_{m-1}|| \leq l_m - l_{m-1} =: \lambda_m$ (measure of discretization errors)

On road-map assisted vehicle tracking

road maps: polygons $\mathcal{R}(l)$ (arc length l, with mapping/discretization errors)

road coordinates: convoy kinematics described by: $\boxed{\mathbf{x}_k^r = (l_k, \dot{l}_k)^\top}$ (2D!)

dynamics: prediction in road coordinates: $p(\mathbf{x}_{k-1}^r|\mathcal{Z}^{k-1}) \xrightarrow{\text{Dyn.}} p(\mathbf{x}_k^r|\mathcal{Z}^{k-1})$

sensor: filtering step in sensor coordinates: $p(\mathbf{x}_k^s|\mathcal{Z}^{k-1}) \xrightarrow[Z_k]{\text{Sen.}} p(\mathbf{x}_k^s|\mathcal{Z}^k)$

$$\underbrace{p(\mathbf{x}_k^r|\mathcal{Z}^{k-1})}_{\text{road coordinates}} \quad \xrightarrow[\text{mapping errors } \mathbf{R}_i^j]{\text{road network}} \quad \underbrace{p(\mathbf{x}_k^s|\mathcal{Z}^{k-1})}_{\text{sensor coordinates}}$$

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- **result: GAUSSIAN mixtures referring to road segments**
- **road segments: essentially artificial scalar measurements**
- **in principle seamlessly embedded into BAYES*ian formalism***
- **road *networks* imply an inherent multihypothesis structure**