Introduction to Sensor Data Fusion Methods and Applications

- **Last lecture: Why Sensor Data Fusion?**
  - Motivation, general context
  - Discussion of examples
- **Today: Steep climb to a first algorithm.**

- **oral examination: 6 credit points after the end of the semester**
- **prerequisite: participate in the excercises, explain a good program**
- **job opportunities as research assistant in ongoing projects, practicum**
- **subsequently: bachelor at Fraunhofer FKIE, master / PhD possible**
- **slides/script: email to wolfgang.koch@fkie.fraunhofer.de, download**

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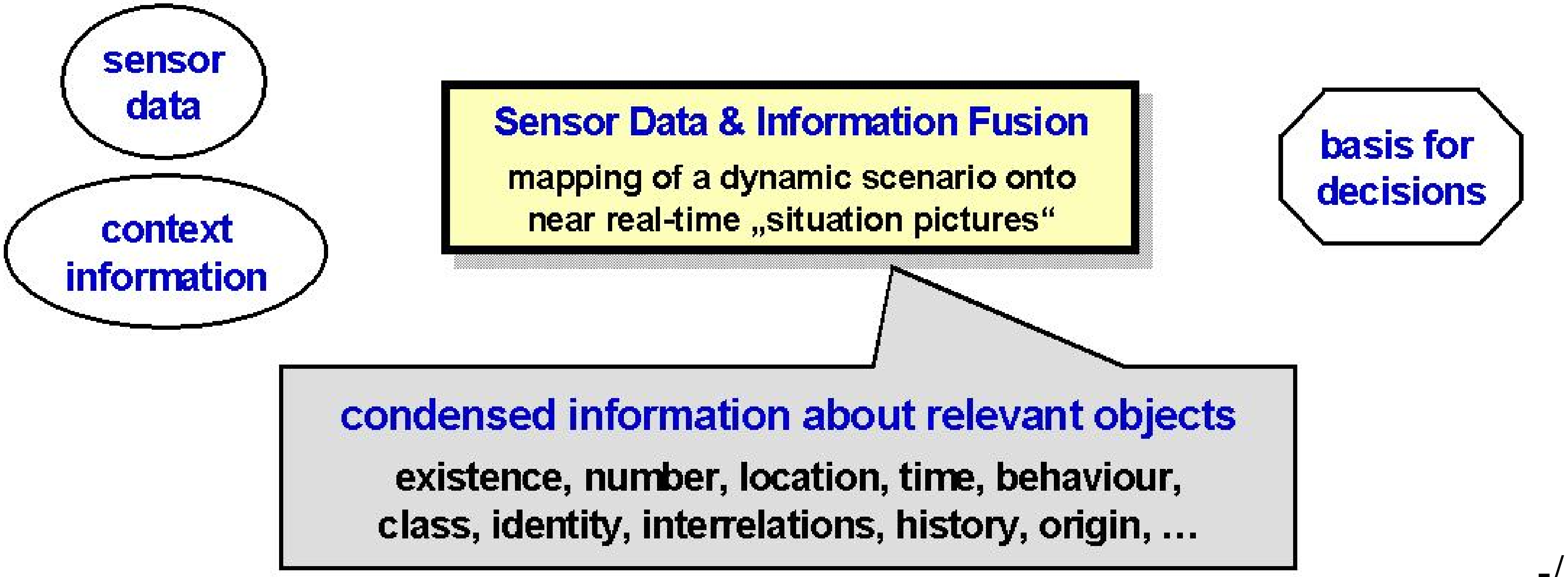
Sensor & Information Fusion: Basic Task



information sources: defined by operational requirements

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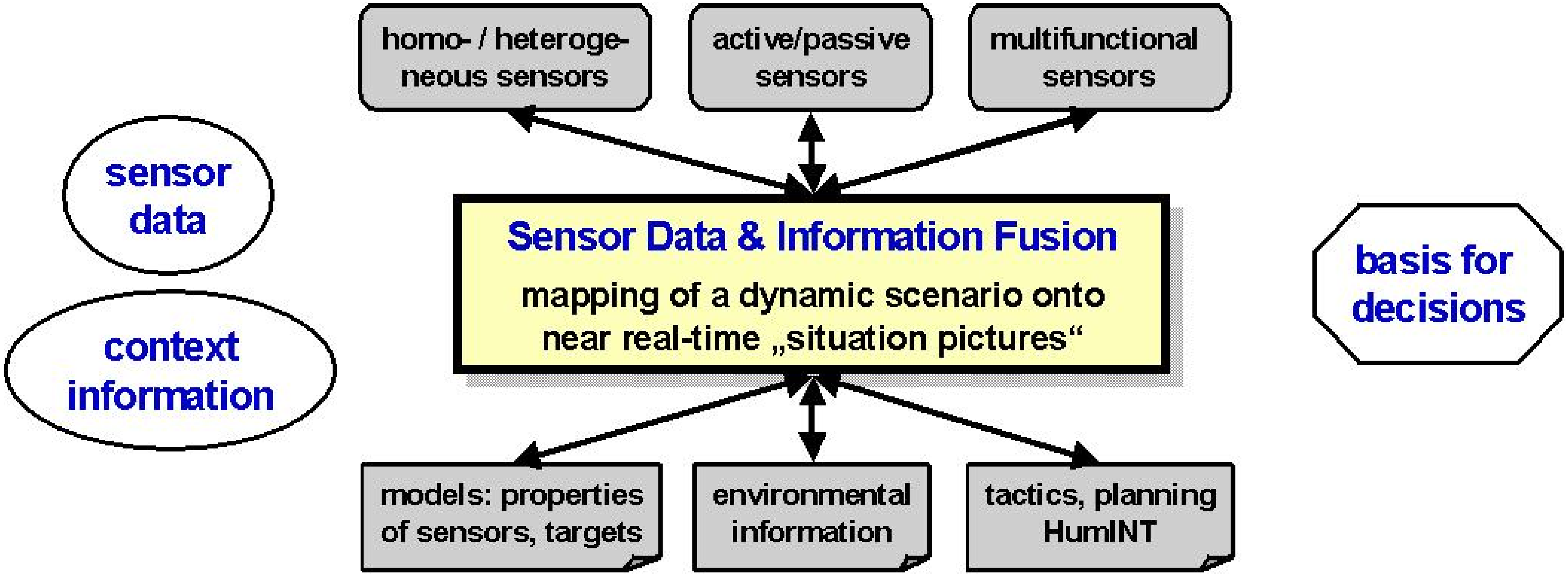
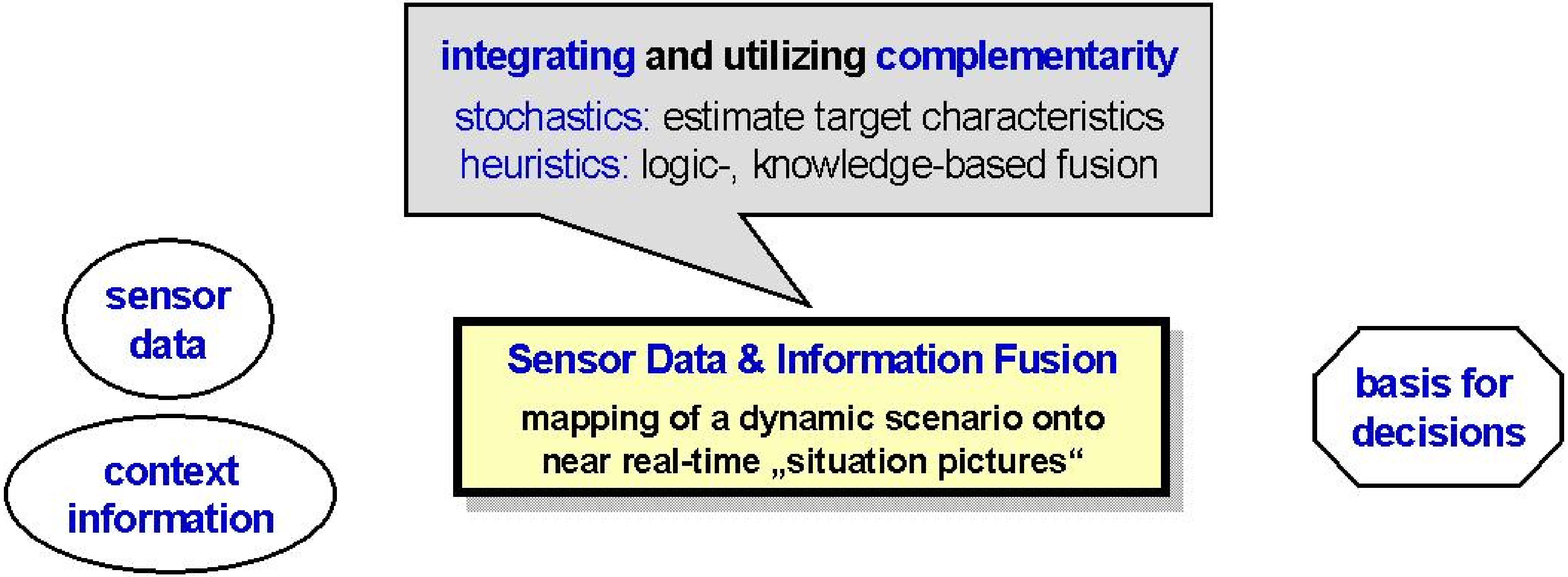
Sensor & Information Fusion: Basic Task



information to be fused: imprecise, incomplete, ambiguous, un-resolved, false, deceptive, hard to formalize, contradictory . . .

Sensor & Information Fusion: Basic Task



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information sources: defined by operational requirements

Target Tracking: Basic Idea, Demonstration

Problem-inherent uncertainties and ambiguities!

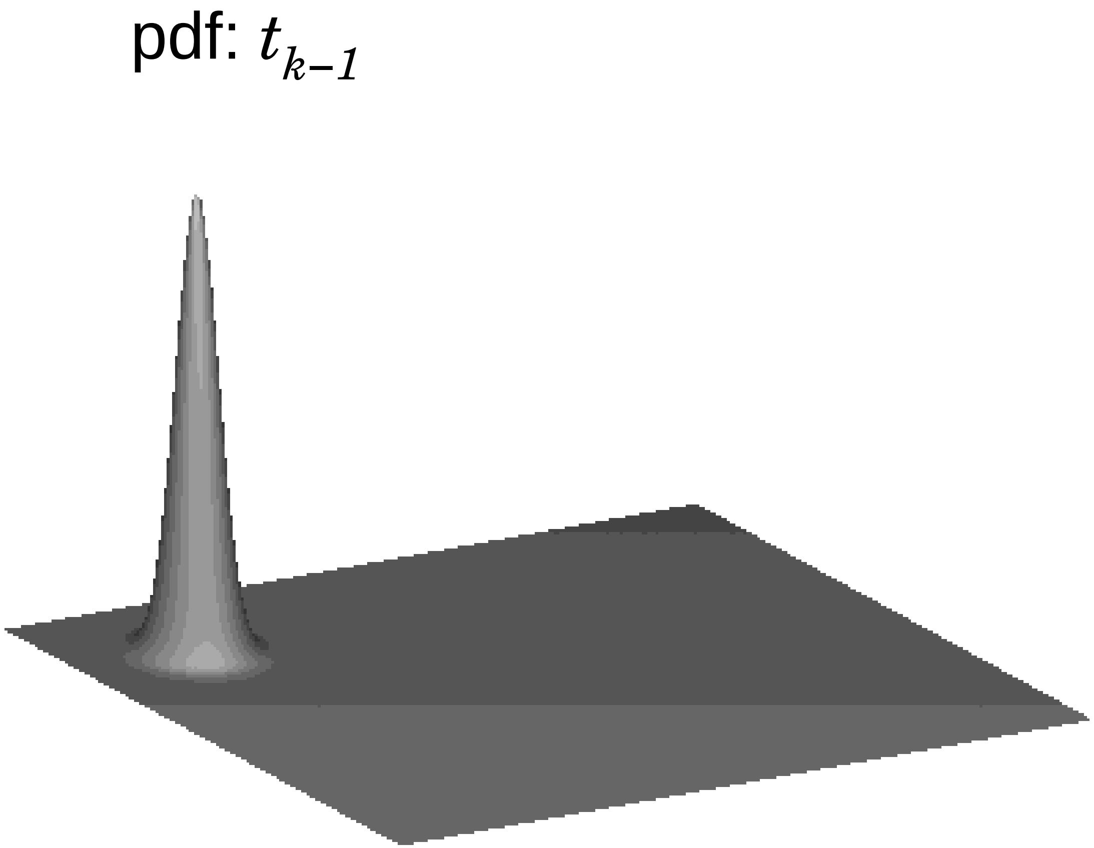
BAYES: processing scheme for *'soft'*, *'delayed'* decision

sensor performance: • resolution conflicts • DOPPLER blindness

environment: • dense situations • clutter • jamming/deception

target characteristics: • qualitatively distinct maneuvering phases

background knowledge • vehicles on road *networks* • behavior



‘Probability densities functions (pdf)’ $p(\mathrm{x}_{k-1}|\mathcal{Z}^{k-1})$ **represent imprecise knowledge on the ‘state’** x_{k-1} **based on imprecise measurements** $\mathcal{Z}^{k-1}$**.**

Characterize an object by *quantitatively describable* properties: object state

Examples:

- object position x on a strait line: $x \in \mathbb{R}$
- kinematic state $\mathbf{x} = (\mathbf{r}^\top, \dot{\mathbf{r}}^\top, \ddot{\mathbf{r}}^\top)^\top$, $\mathbf{x} \in \mathbb{R}^9$

  position $\mathbf{r} = (x, y, z)^\top$, velocity $\dot{\mathbf{r}}$, acceleration $\ddot{\mathbf{r}}$
- joint state of two objects: $\mathbf{x} = (\mathbf{x}_1^\top, \mathbf{x}_2^\top)^\top$
- kinematic state $\mathbf{x}$, object extension $\mathbf{X}$

  z.B. ellipsoid: symmetric, positively definite matrix
- kinematic state $\mathbf{x}$, object class class

  z.B. bird, sailing plane, helicopter, passenger jet, ...

Learn unknown object states from imperfect measurements and
describe by functions $p(\mathbf{x})$ imprecise knowledge mathematically precisely!

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How to deal with probability density functions?

- pdf $p(x)$: Extract *probability statements* about the RV x by integration!
- naïvely: *positive* and *normalized* functions ($p(x) \geq 0$, $\int dx\, p(x) = 1$)

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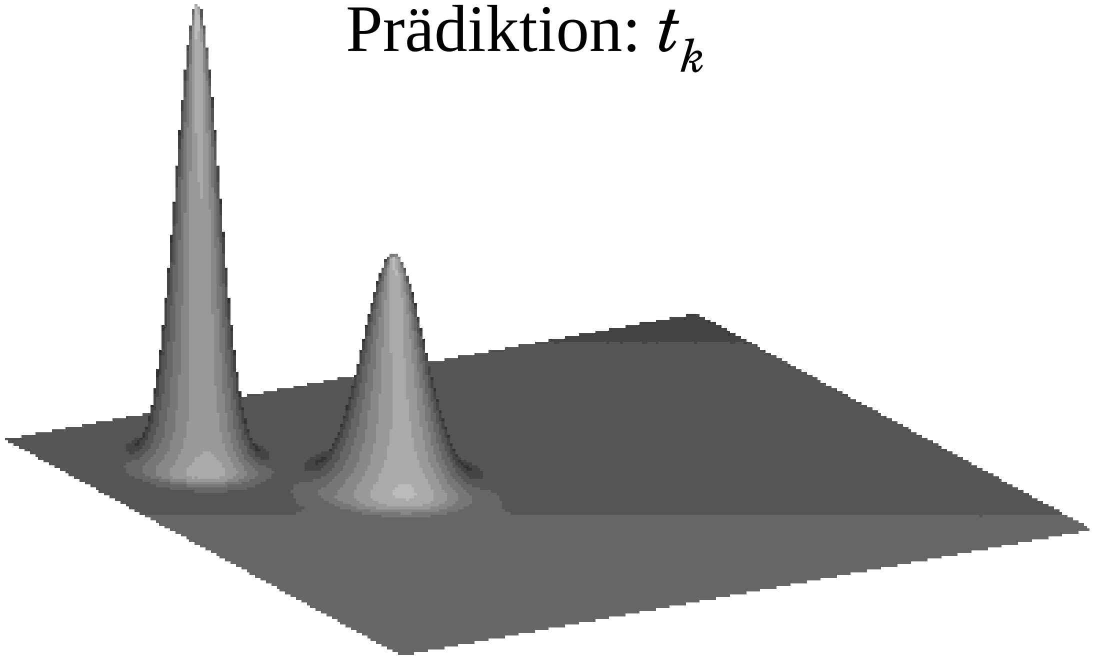

Exploit imprecise knowledge on the dynamical behavior of the object.

$$\underbrace{p(\mathbf{x}_k|\mathcal{Z}^{k-1})}_{\text{prediction}} = \int d\mathbf{x}_{k-1}\ \underbrace{p(\mathbf{x}_k|\mathbf{x}_{k-1})}_{\text{dynamics}}\ \underbrace{p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1})}_{\text{old knowledge}}.$$

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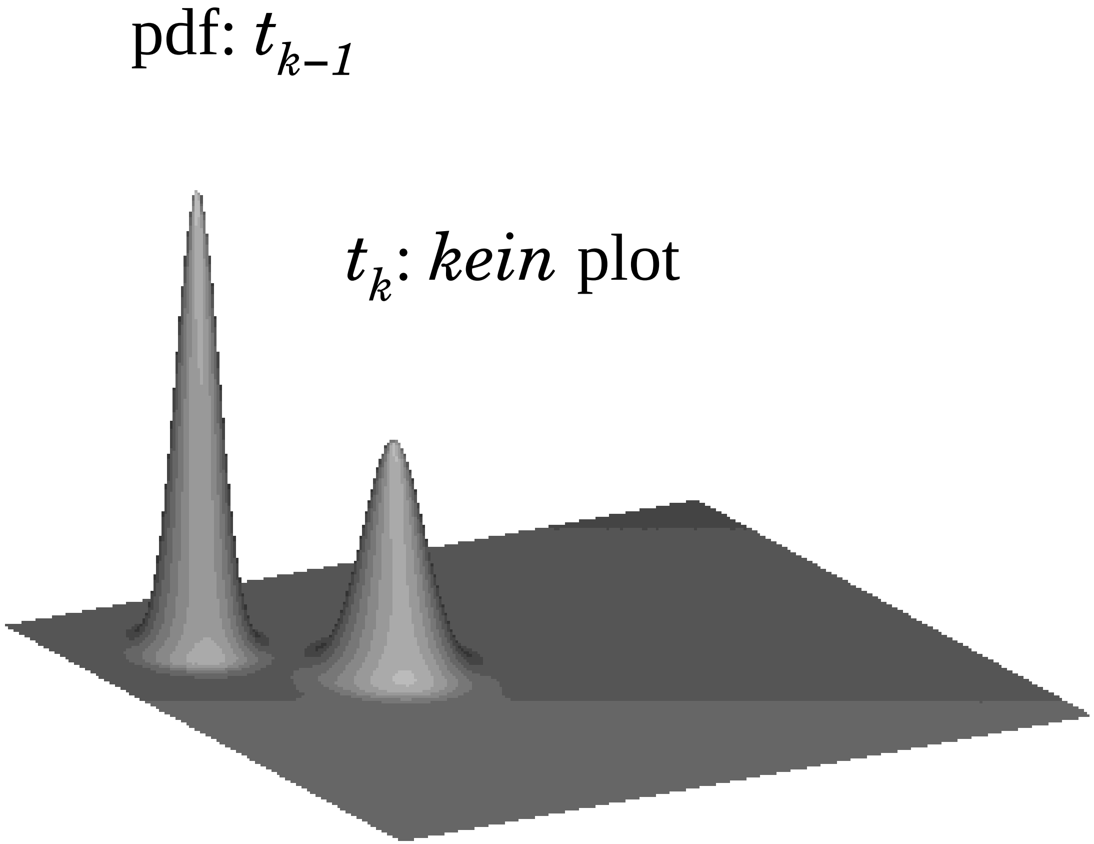
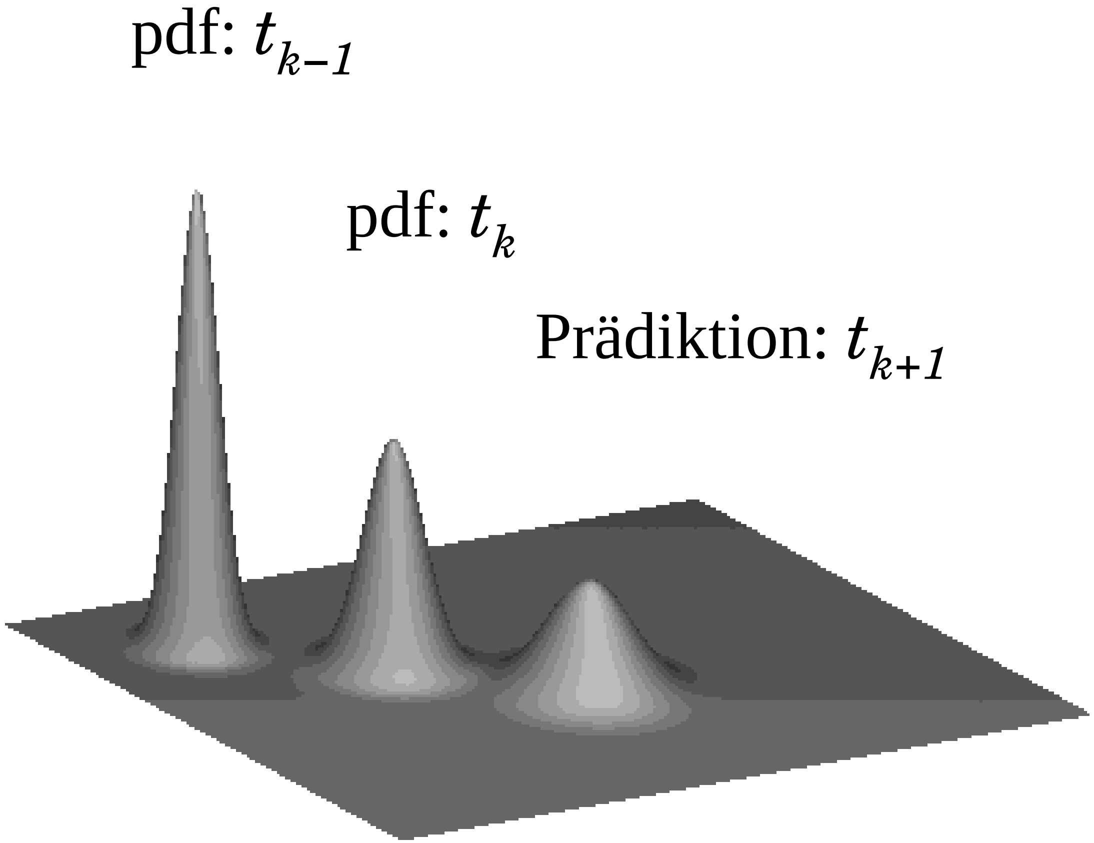
- *conditional pdf* $p(x|y) = \frac{p(x,y)}{p(y)}$: Impact of information on y on RV x?
- *marginal density* $p(x) = \int dy\, p(x,y) = \int dy\, p(x|y)\, p(y)$: Enter y!

$$
\begin{aligned}
p(\mathbf{x}_k|\mathcal{Z}^{k-1}) &= \int d\mathbf{x}_{k-1}\, p(\mathbf{x}_k, \mathbf{x}_{k-1}|\mathcal{Z}^{k-1}) \\
&= \int d\mathbf{x}_{k-1}\, p(\mathbf{x}_k|\mathbf{x}_{k-1}, \mathcal{Z}^{k-1})\, p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1}) \\
&= \int d\mathbf{x}_{k-1}\, p(\mathbf{x}_k|\mathbf{x}_{k-1})\, p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1})
\end{aligned}
$$

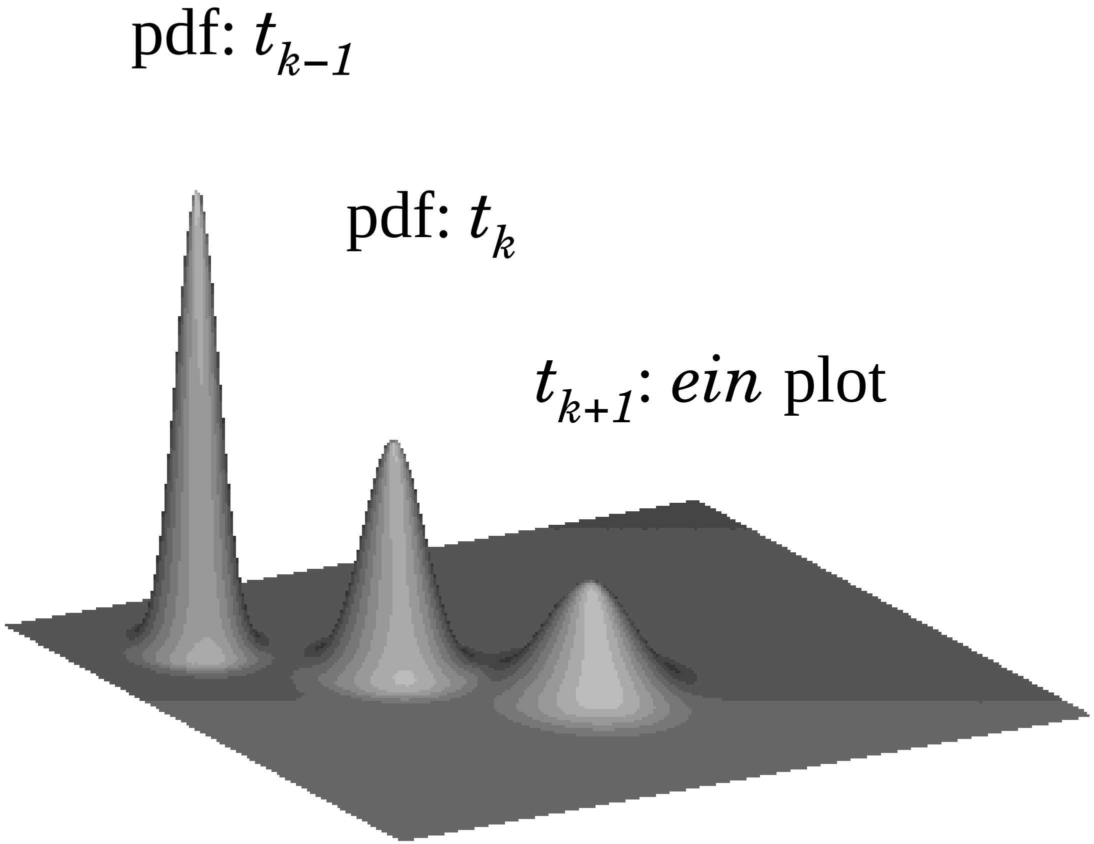


missing sensor detection: ‘data processing’ = prediction

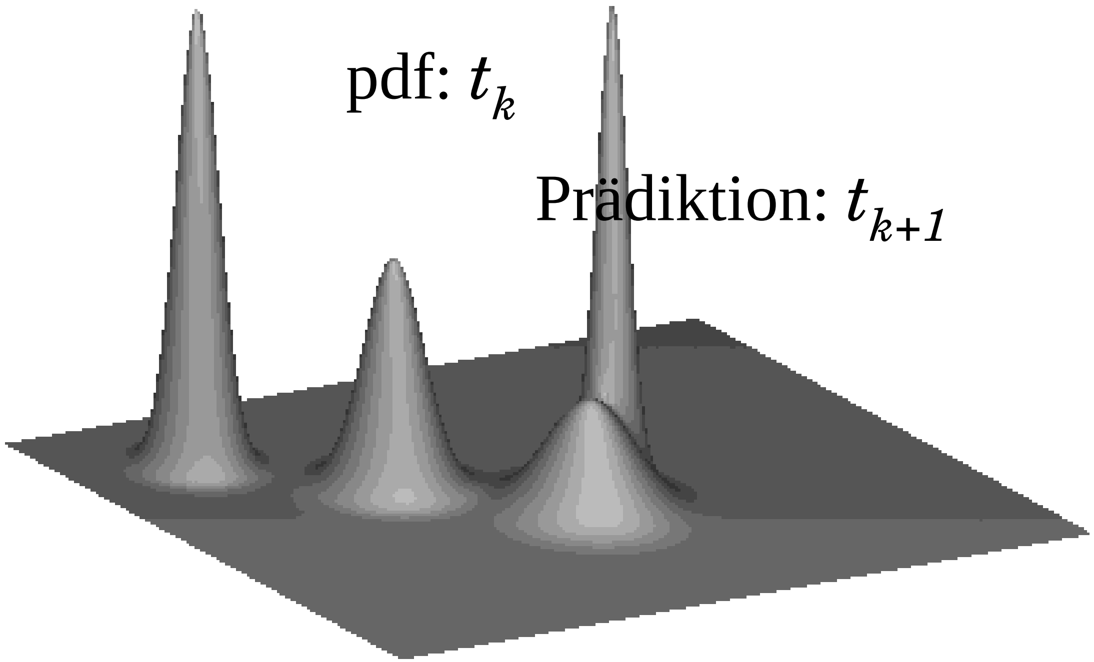
(not always: exploitation of ‘negative’ sensor evidence)



missing sensor information: increasing knowledge dissipation



sensor information on the kinematical object state



BAYES' formula: $$\underbrace{p(\mathbf{x}_{k+1}|\mathcal{Z}^{k+1})}_{\text{new knowledge}} = \frac{p(\mathbf{z}_{k+1}|\mathbf{x}_{k+1})\, p(\mathbf{x}_{k+1}|\mathcal{Z}^k)}{\int d\mathbf{x}_{k+1}\, p(\underbrace{\mathbf{z}_{k+1}}_{\text{plot}}|\mathbf{x}_{k+1})\, \underbrace{p(\mathbf{x}_{k+1}|\mathcal{Z}^k)}_{\text{prediction}}}$$

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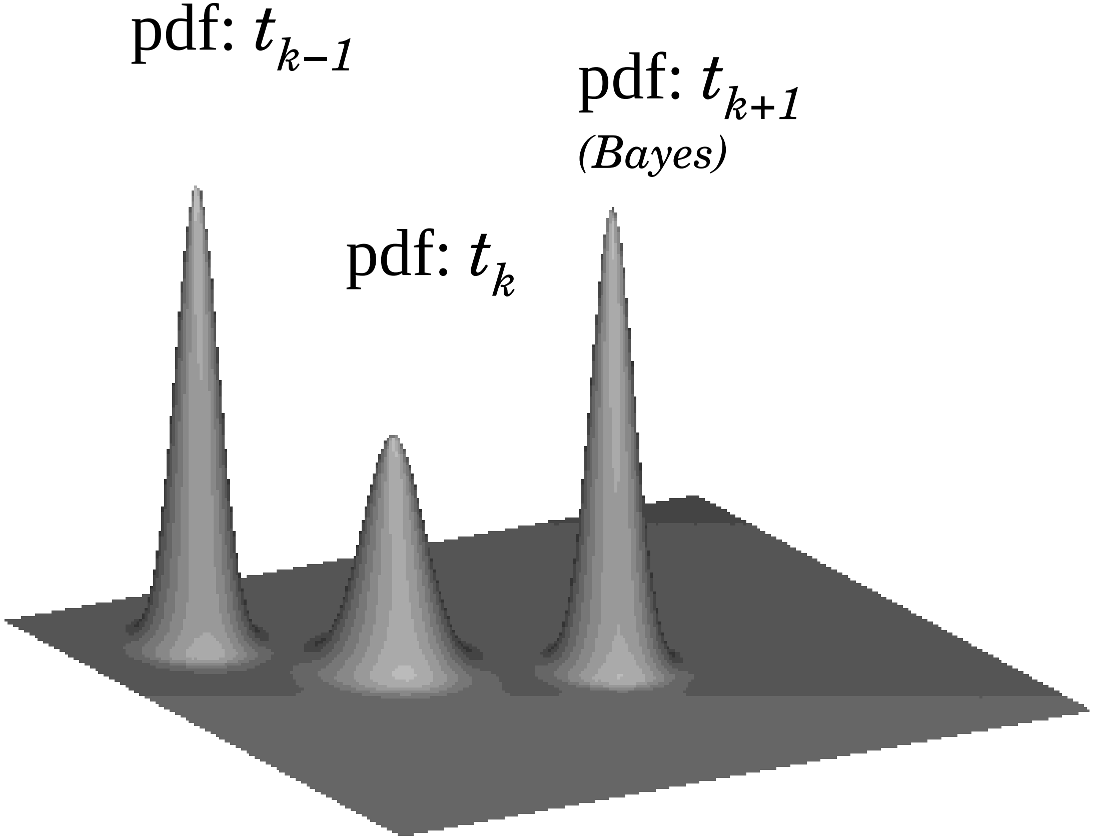
- *marginal density* $p(x) = \int dy\, p(x, y) = \int dy\, p(x|y)\, p(y)$: Enter y!

- Bayes: $p(x|y) = \frac{p(y|x)p(x)}{p(y)} = \frac{p(y|x)p(x)}{\int dx\, p(y|x)p(x)}$: $p(x|y) \leftarrow p(y|x), p(x)$!

$$p(x|y)\, p(y) = p(x, y) = p(y, z) = p(y|x)\, p(x)$$



filtering = sensor data processing

Target or Object Tracking: Basic Idea

Iterative updating of conditional probability densities!

kinematic target state x_k at time t_k, **accumulated sensor data** $\mathcal{Z}^k$

a priori knowledge: target dynamics models, sensor model

- **prediction:** $p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1}) \xrightarrow{\text{dynamics model}} p(\mathbf{x}_k|\mathcal{Z}^{k-1})$
- **filtering:** $p(\mathrm{x}_k|\mathcal{Z}^{k-1}) \xrightarrow[\text{sensor model}]{\text{sensor data } Z_k} p(\mathrm{x}_k|\mathcal{Z}^k)$
- **retrodiction:** $p(\mathbf{x}_{l-1}|\mathcal{Z}^k) \xleftarrow[\text{dynamics model}]{\text{filtering output}} p(\mathbf{x}_l|\mathcal{Z}^k)$

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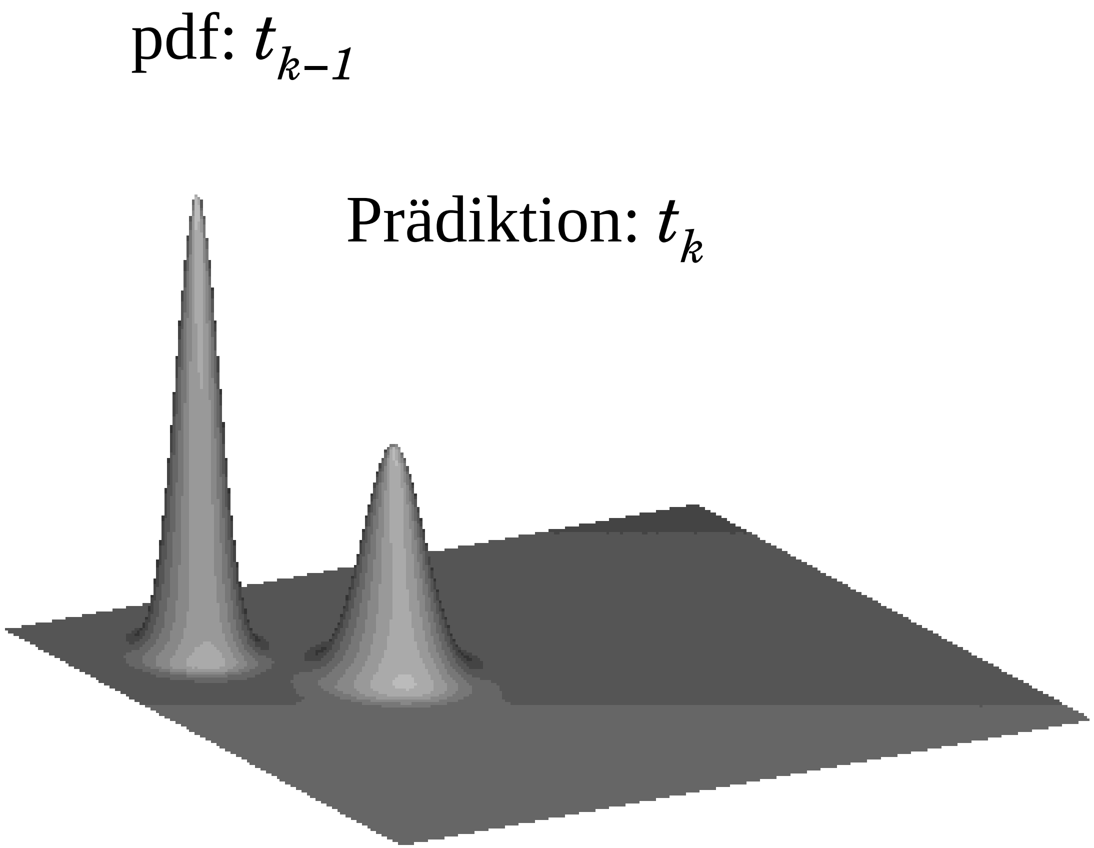

Exploit imprecise knowledge on the dynamical behavior of the object.

$$\underbrace{p(\mathbf{x}_k|\mathcal{Z}^{k-1})}_{\text{prediction}} = \int d\mathbf{x}_{k-1}\ \underbrace{p(\mathbf{x}_k|\mathbf{x}_{k-1})}_{\text{dynamics}}\ \underbrace{p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1})}_{\text{old knowledge}}.$$

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The Multivariate GAUSSian Pdf

- *wanted:* probabilities 'concentrated' around a center $\mathbf{x}$

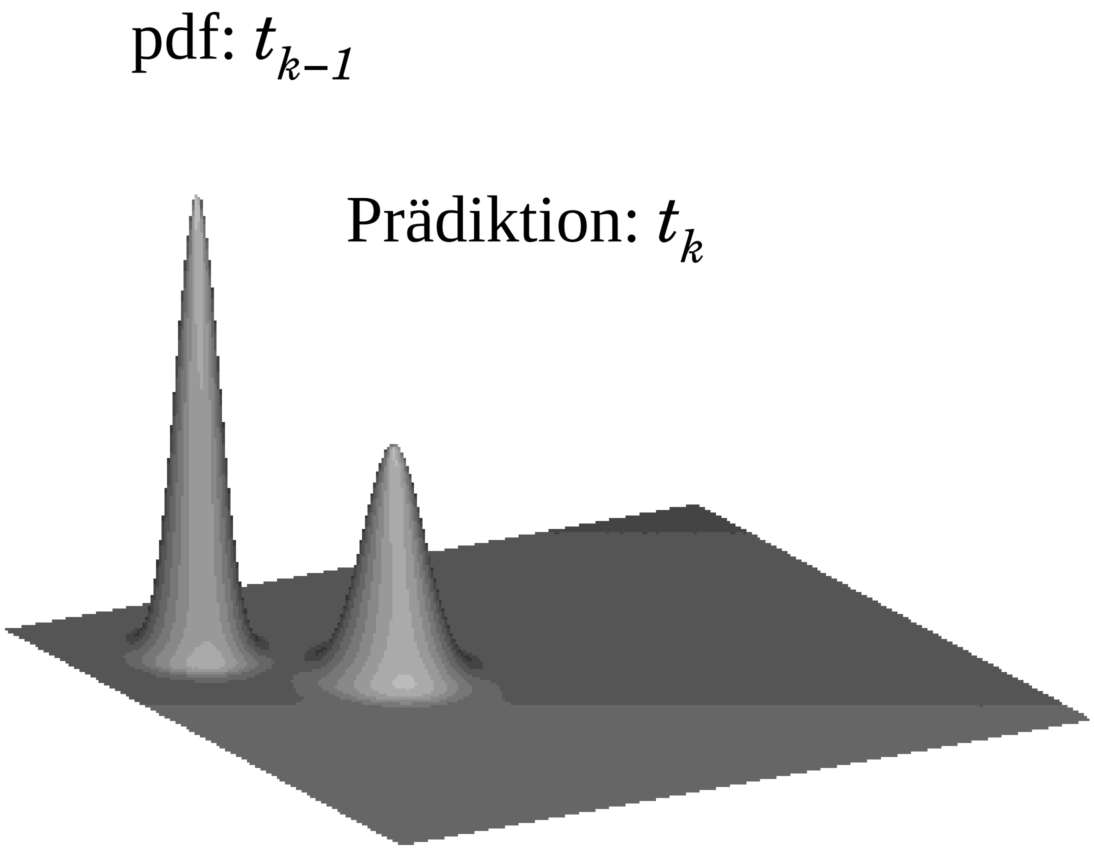
- *quadratic distance:* $q(x) = \frac{1}{2}(x - \mathbf{x})\mathbf{P}^{-1}(x - \mathbf{x})^\top$

  $q(x)$ defines an ellipsoid around $\mathbf{x}$, its volume and orientation being determined by a matrix $\mathbf{P}$ (symmetric: $\mathbf{P}^\top = \mathbf{P}$, positively definite: all eigenvalues > 0).

- *first attempt:* $p(x) = \mathrm{e}^{-q(x)} / \int dx\, \mathrm{e}^{-q(x)}$ (normalized!)

$$p(x) = \mathcal{N}(x;\, \mathbf{x},\, \mathbf{P}) = \frac{1}{\sqrt{|2\pi\mathbf{P}|}}\, \mathrm{e}^{-\frac{1}{2}(x-\mathbf{x})^\top\mathbf{P}^{-1}(x-\mathbf{x})}$$

- GAUSS*ian Mixtures:* $p(x) = \sum_i p_i\, \mathcal{N}(x;\, \mathbf{x}_i,\, \mathbf{P}_i)$ (weighted sums)



Exploit imprecise knowledge on the dynamical behavior of the object.

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A Useful Product Formula for Gaussians

$$\mathcal{N}(\mathbf{z};\, \mathbf{F}\mathbf{x},\, \mathbf{D})\; \mathcal{N}(\mathbf{x};\, \mathbf{y},\, \mathbf{P}) = \underbrace{\mathcal{N}(\mathbf{z};\, \mathbf{F}\mathbf{y},\, \mathbf{S})}_{\text{independent of } \mathbf{x}}\; \mathcal{N}(\mathbf{x};\, \mathbf{y} + \mathbf{W}\boldsymbol{\nu},\, \mathbf{P} - \mathbf{W}\mathbf{S}\mathbf{W}^\top)$$

$$\boldsymbol{\nu} = \mathbf{z} - \mathbf{F}\mathbf{y}, \quad \mathbf{S} = \mathbf{F}\mathbf{P}\mathbf{F}^\top + \mathbf{D}, \quad \mathbf{W} = \mathbf{P}\mathbf{F}^\top\mathbf{S}^{-1}.$$

Kalman filter: $\mathbf{x}_k = (\mathbf{r}_k^\top, \dot{\mathbf{r}}_k^\top)^\top$, $\mathcal{Z}^k = \{\mathbf{z}_k, \mathcal{Z}^{k-1}\}$

initiation: $p(\mathbf{x}_0) = \mathcal{N}(\mathbf{x}_0;\, \mathbf{x}_{0|0},\, \mathbf{P}_{0|0})$, initial ignorance: $\mathbf{P}_{0|0}$ 'large'

prediction: $\mathcal{N}(\mathbf{x}_{k-1};\, \mathbf{x}_{k-1|k-1},\, \mathbf{P}_{k-1|k-1}) \xrightarrow[\mathbf{F}_{k|k-1},\, \mathbf{D}_{k|k-1}]{\text{dynamics model}} \mathcal{N}(\mathbf{x}_k;\, \mathbf{x}_{k|k-1},\, \mathbf{P}_{k|k-1})$

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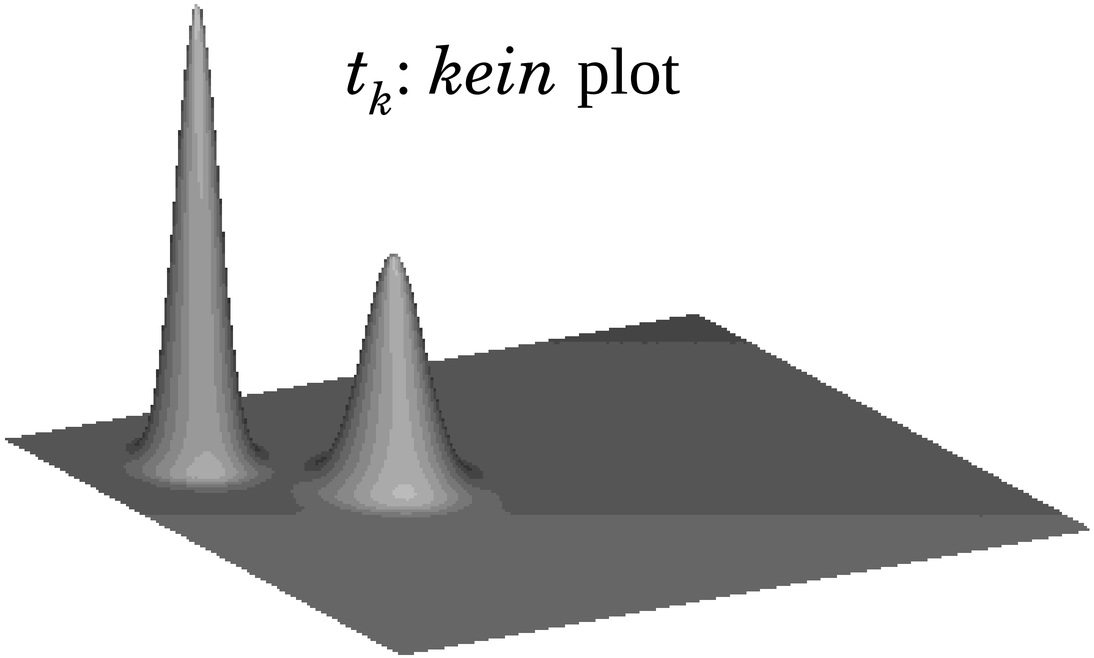
pdf: t_{k-1}

t_k: $kein$ plot



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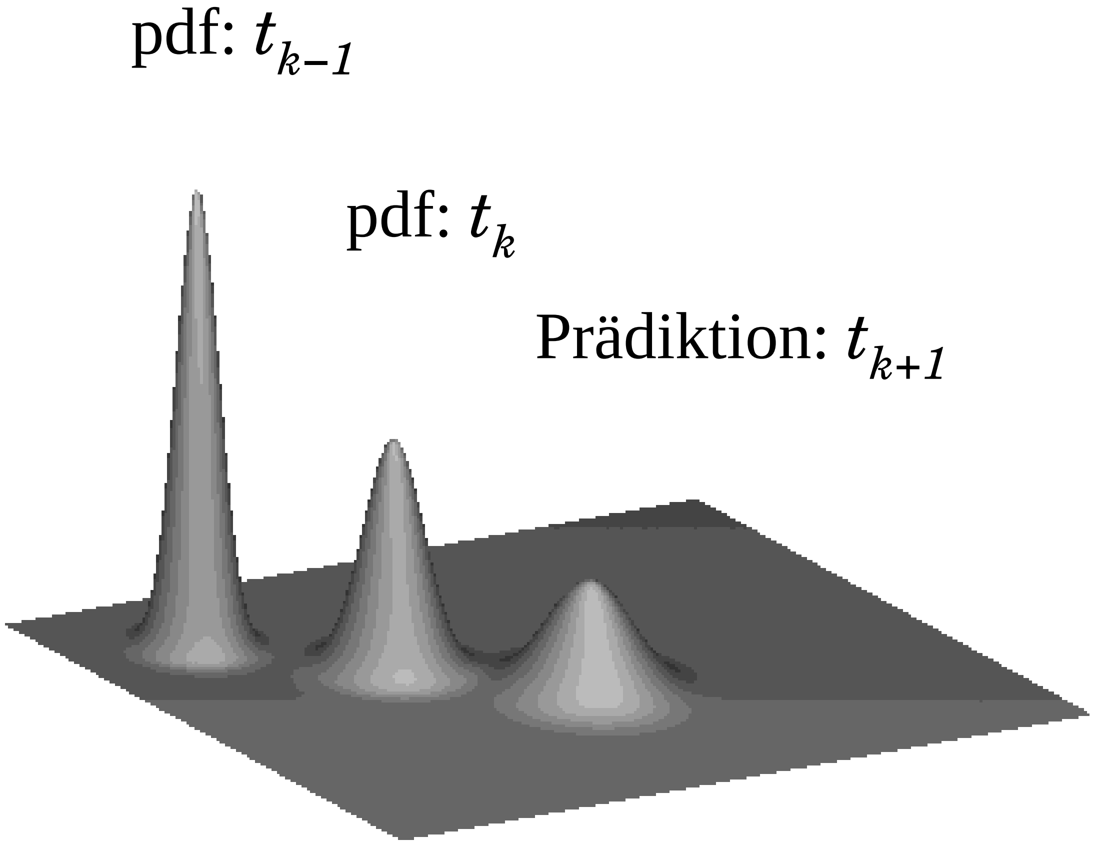

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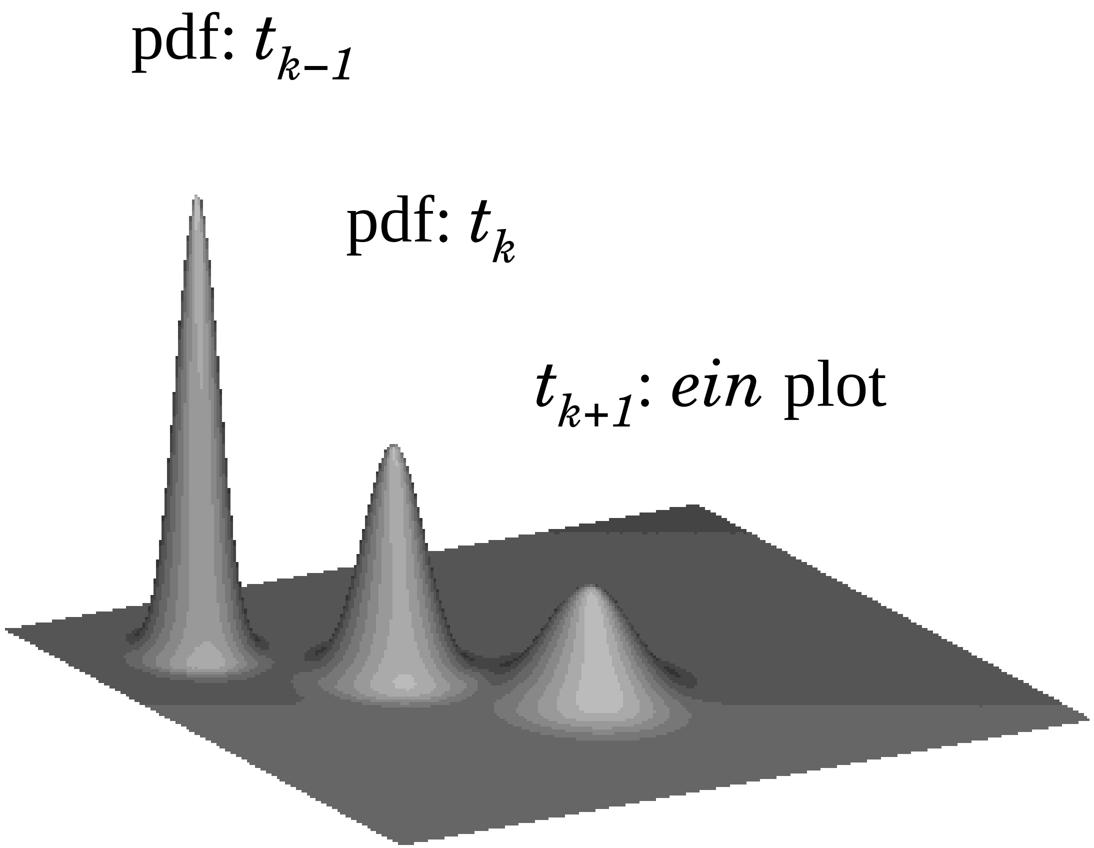

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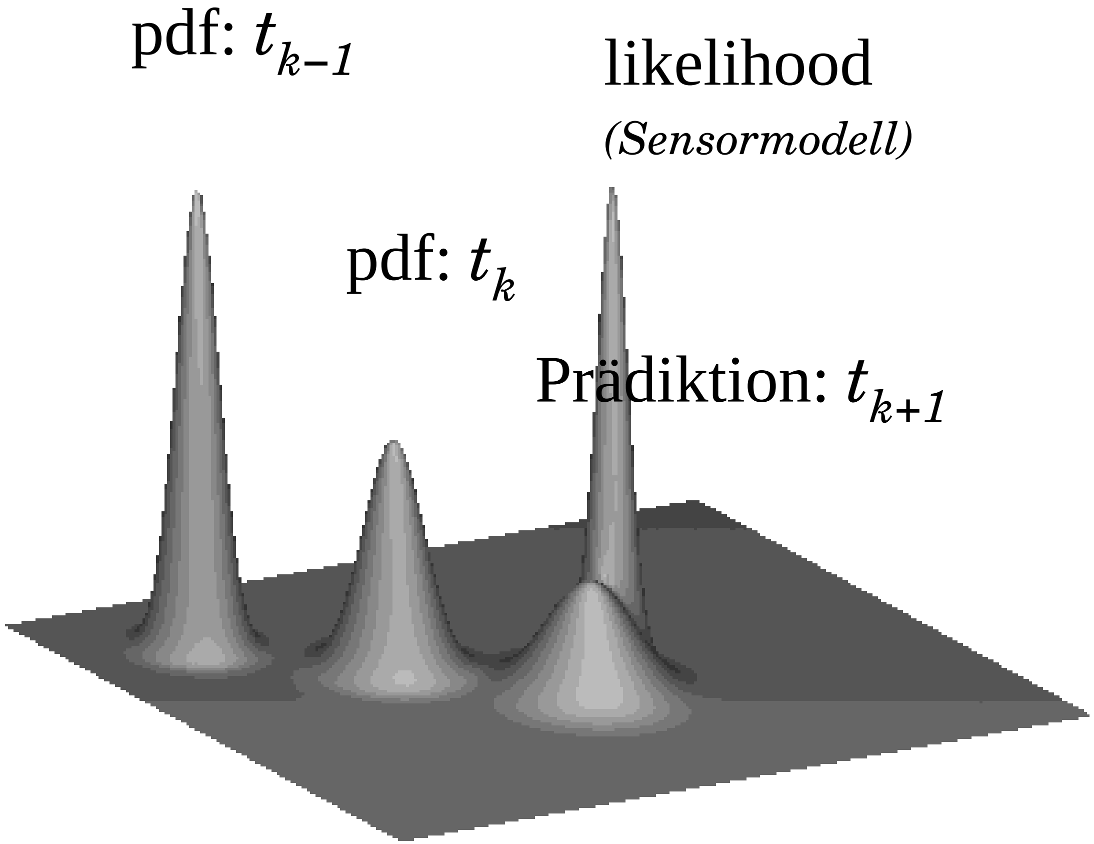

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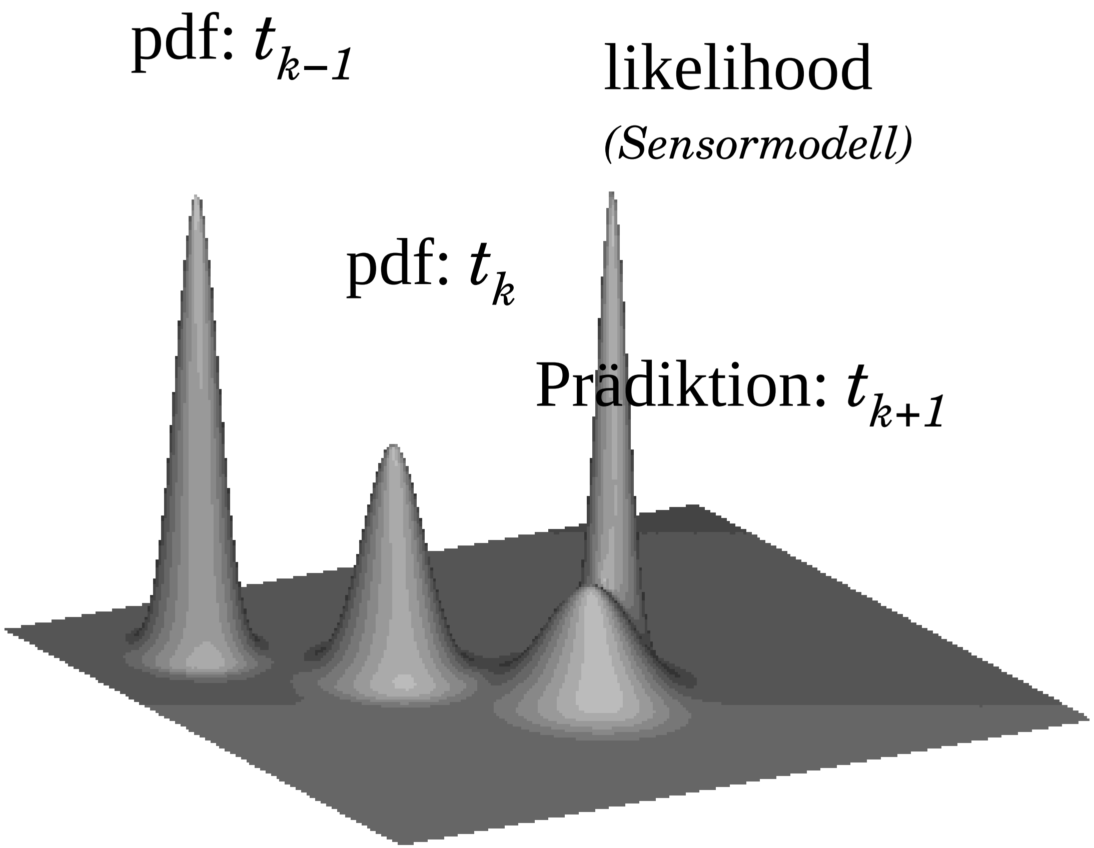

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$$\underbrace{p(\mathbf{x}_{k+1}|\mathcal{Z}^{k+1})}_{\text{new knowledge}} = \frac{\mathcal{N}\big(\mathbf{z}_k;\ \mathbf{H}\mathbf{x}_k,\ \mathbf{R}\big)\ \mathcal{N}\big(\mathbf{x}_k;\ \mathbf{x}_{k|k-1},\ \mathbf{P}_{k|k-1}\big)}{\int d\mathbf{x}_{k+1}\ \mathcal{N}\big(\mathbf{z}_k;\ \mathbf{H}\mathbf{x}_k,\ \mathbf{R}\big)\ \underbrace{\mathcal{N}\big(\mathbf{x}_k;\ \mathbf{x}_{k|k-1},\ \mathbf{P}_{k|k-1}\big)}_{\text{prediction}}}$$

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filtering: $\mathcal{N}(\mathbf{x}_k;\, \mathbf{x}_{k|k-1},\, \mathbf{P}_{k|k-1}) \xrightarrow[\text{sensor model: } \mathbf{H}_k,\, \mathbf{R}_k]{\text{current measurement } \mathbf{z}_k} \mathcal{N}(\mathbf{x}_k;\, \mathbf{x}_{k|k},\, \mathbf{P}_{k|k})$

$$\mathbf{x}_{k|k} = \mathbf{x}_{k|k-1} + \mathbf{W}_{k|k-1}\boldsymbol{\nu}_{k|k-1}, \qquad \boldsymbol{\nu}_{k|k-1} = \mathbf{z}_k - \mathbf{H}_k\mathbf{x}_{k|k-1}$$
$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{W}_{k|k-1}\mathbf{S}_{k|k-1}\mathbf{W}_{k|k-1}{}^\top, \qquad \mathbf{S}_{k|k-1} = \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k{}^\top + \mathbf{R}_k$$
$$\mathbf{W}_{k|k-1} = \mathbf{P}_{k|k-1}\mathbf{H}_k{}^\top\mathbf{S}_{k|k-1}{}^{-1}$$

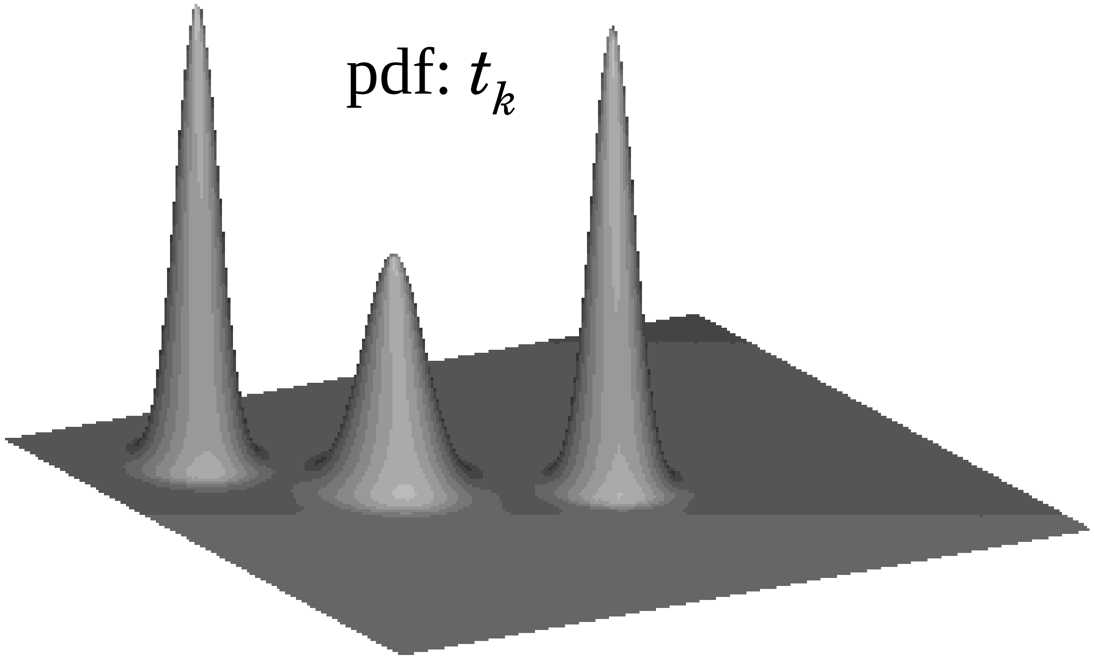
'KALMAN gain matrix'

pdf: t_{k-1}

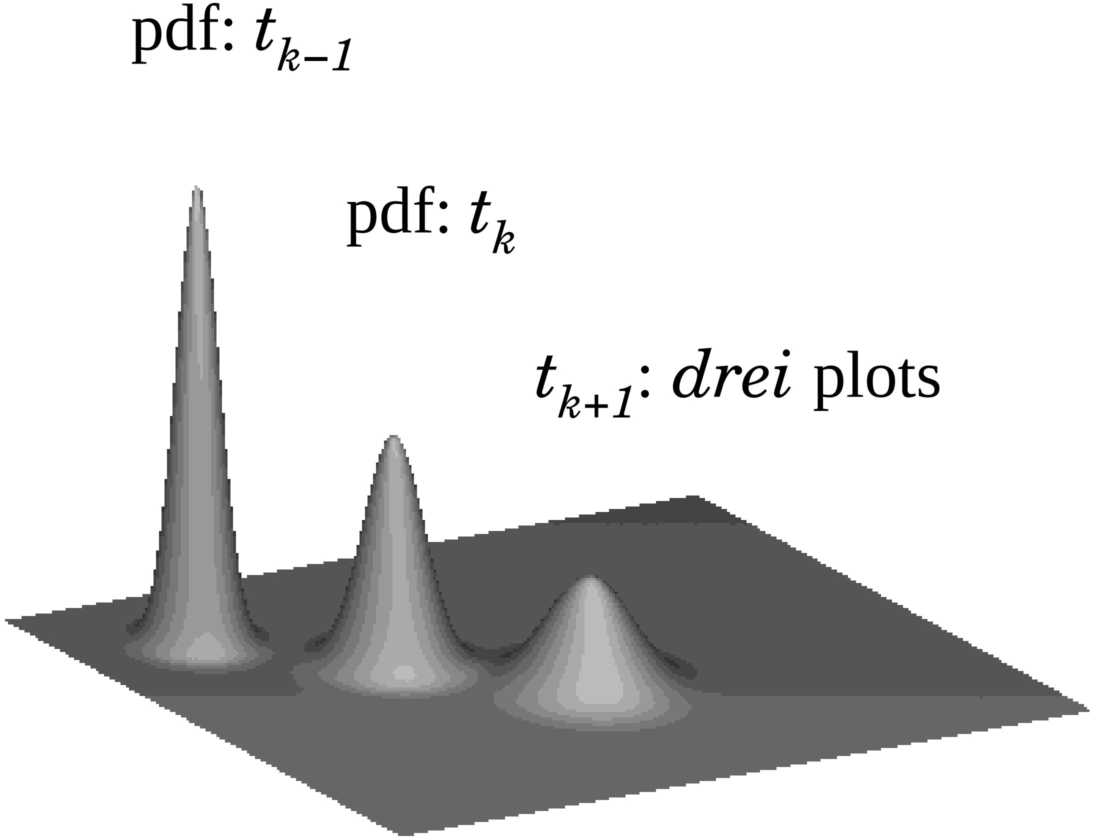
pdf: t_{k+1}

(Bayes)

pdf: t_k



filtering = sensor data processing



ambiguities by false plots: 1 + 3 data interpretation hypotheses

(‘detection probability’, false alarm statistics)

The Multivariate GAUSSian Pdf

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– *quadratic distance:* $q(x) = \frac{1}{2}(x - \mathbf{x})\mathbf{P}^{-1}(x - \mathbf{x})^\top$

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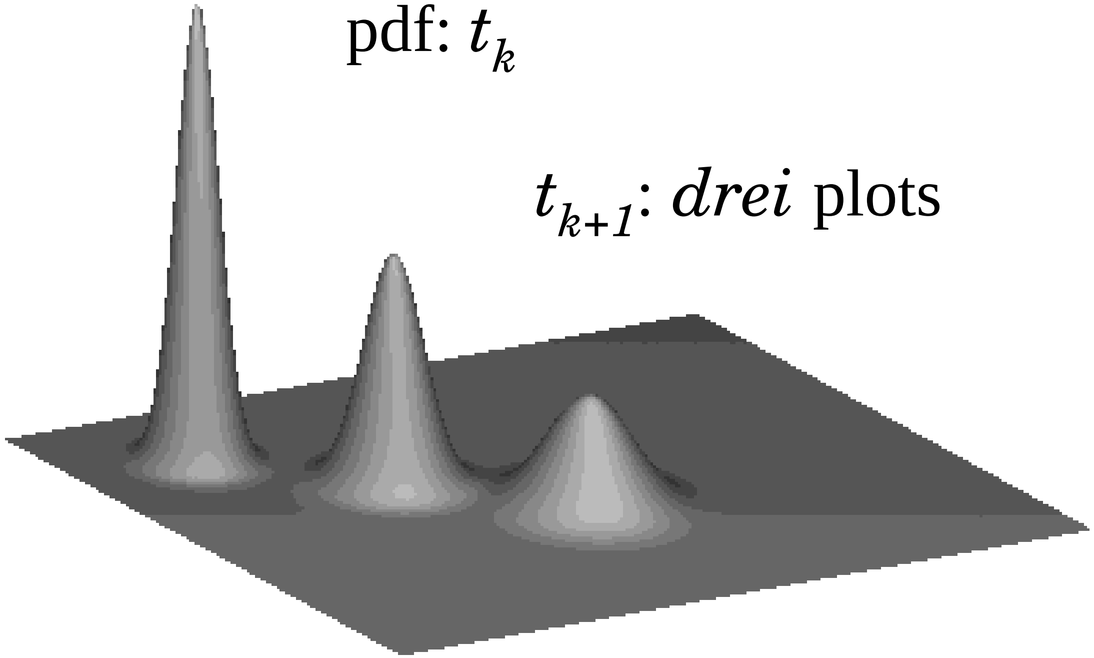
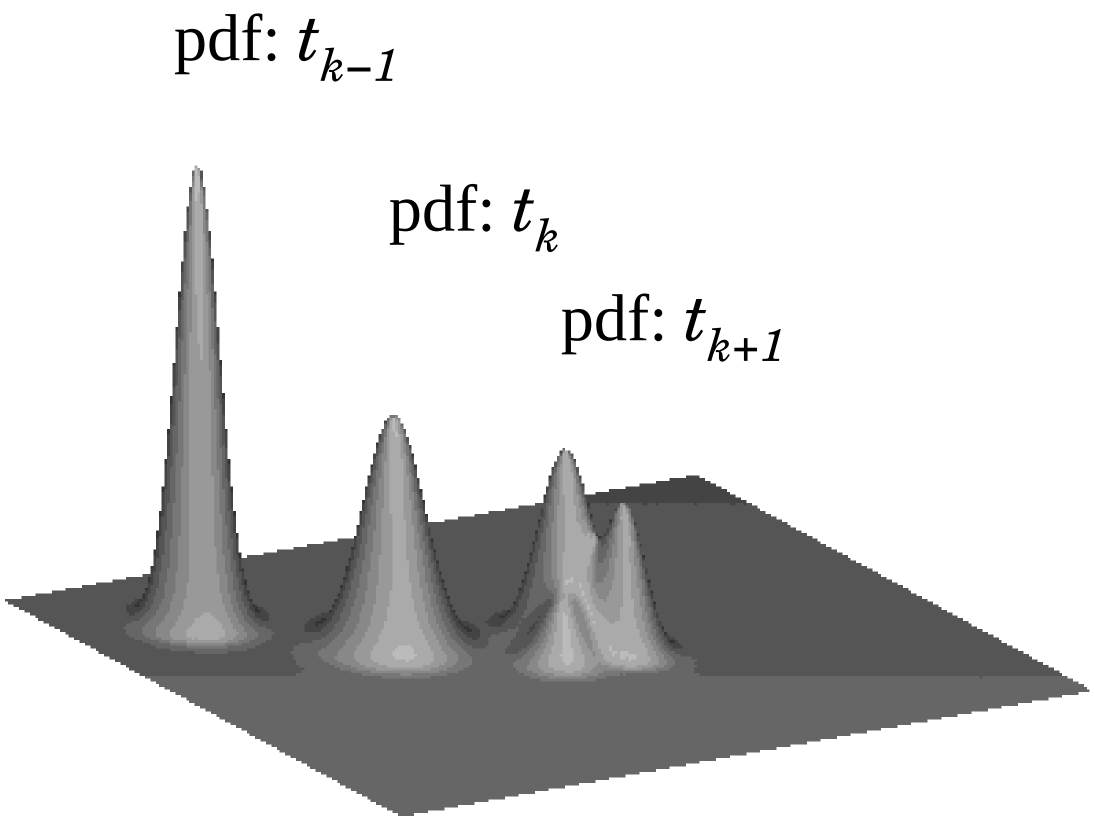
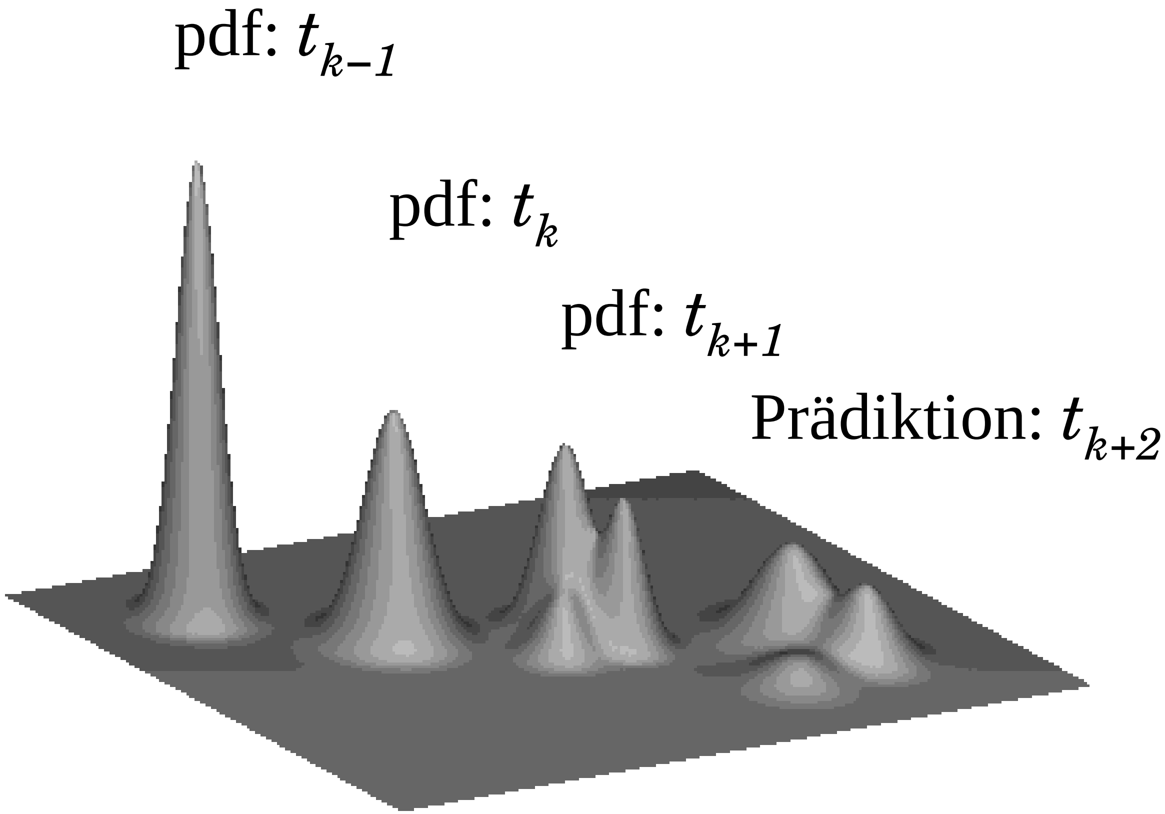
– GAUSS*ian Mixtures:* $p(x) = \sum_i p_i\, \mathcal{N}(x;\, \mathbf{x}_i,\, \mathbf{P}_i)$ (weighted sums)



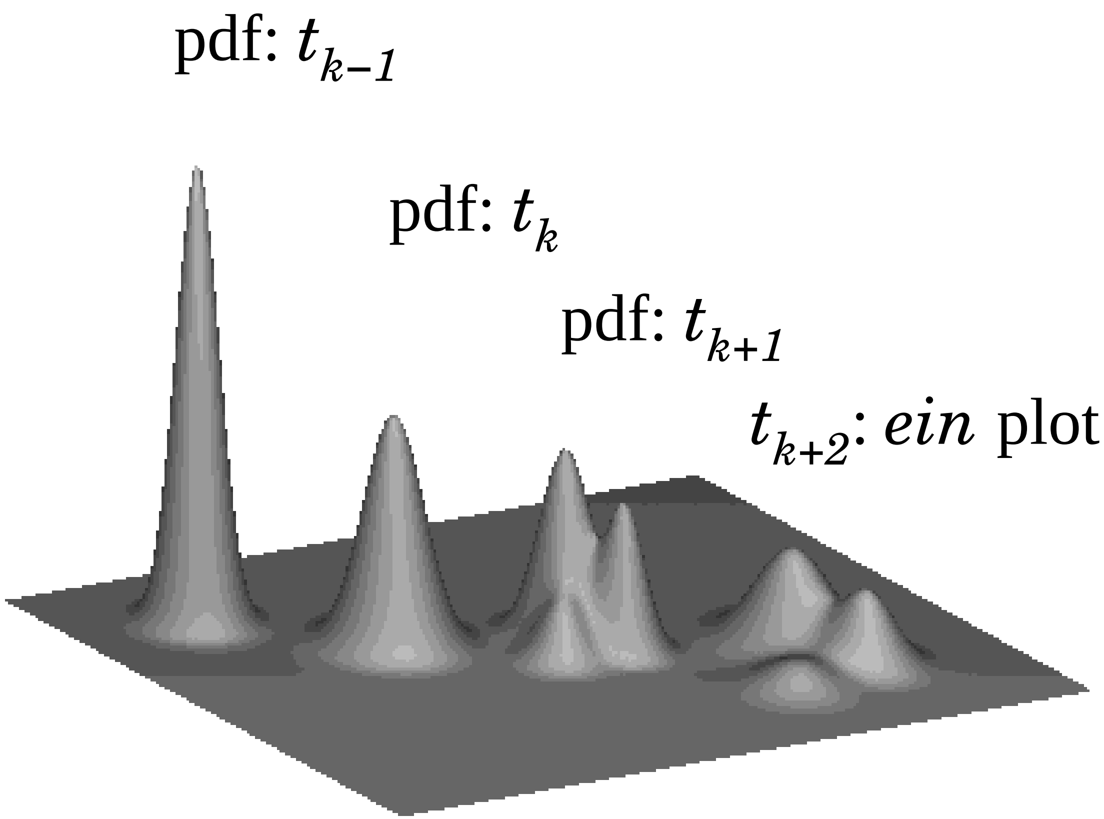
$$p(Z_k, m_k|\mathbf{x}_k) = \text{const.} \left((1 - P_D)\rho_F + P_D \sum_{j=1}^{m_k} \mathcal{N}\Big(\mathbf{z}_k^j;\, \mathbf{H}\mathbf{x}_k,\, \mathbf{R}_k^j\Big) \right)$$



Multimodal pdfs reflect ambiguities inherent in the data.



temporal propagation: dissipation of the probability densities



association tasks: sensor data $\leftrightarrow$ interpretation hypotheses

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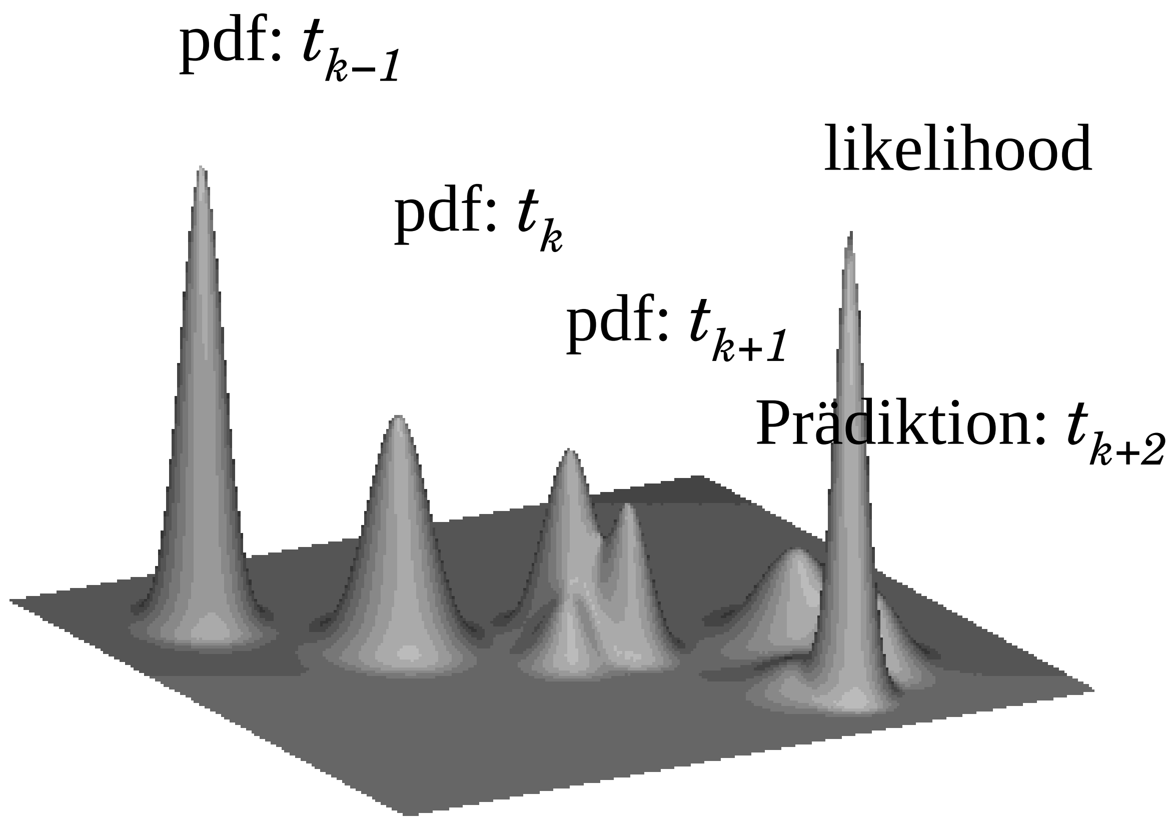
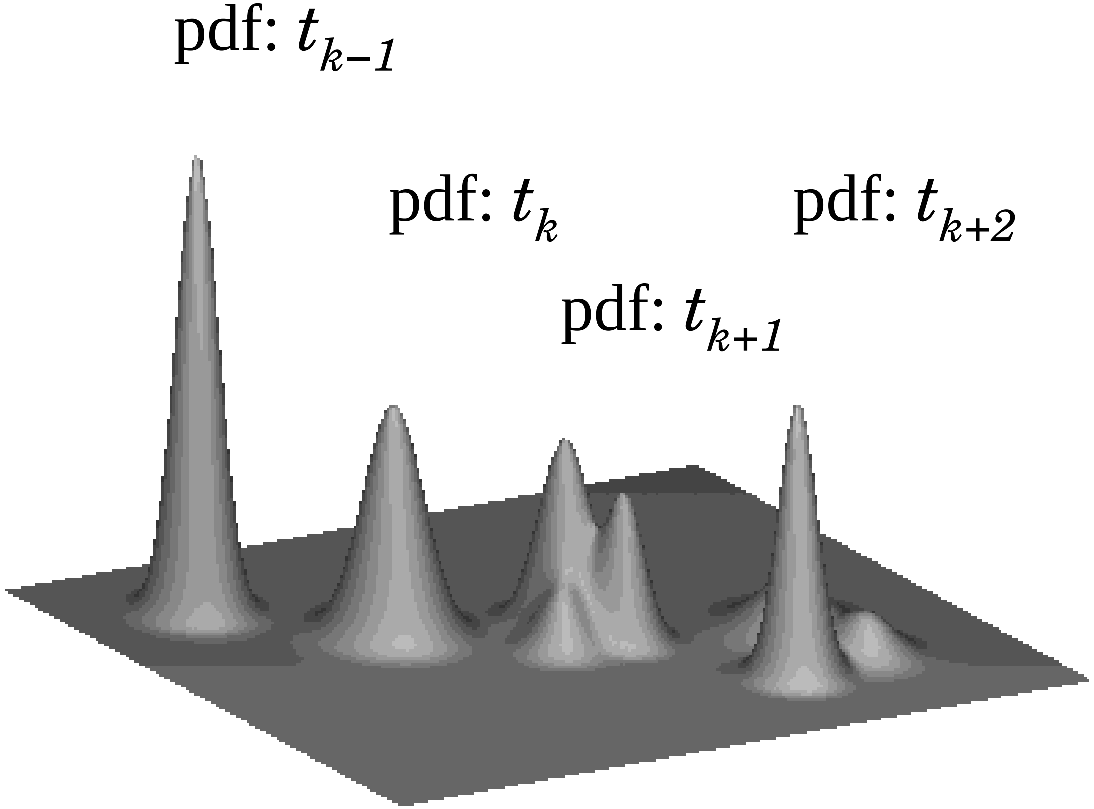
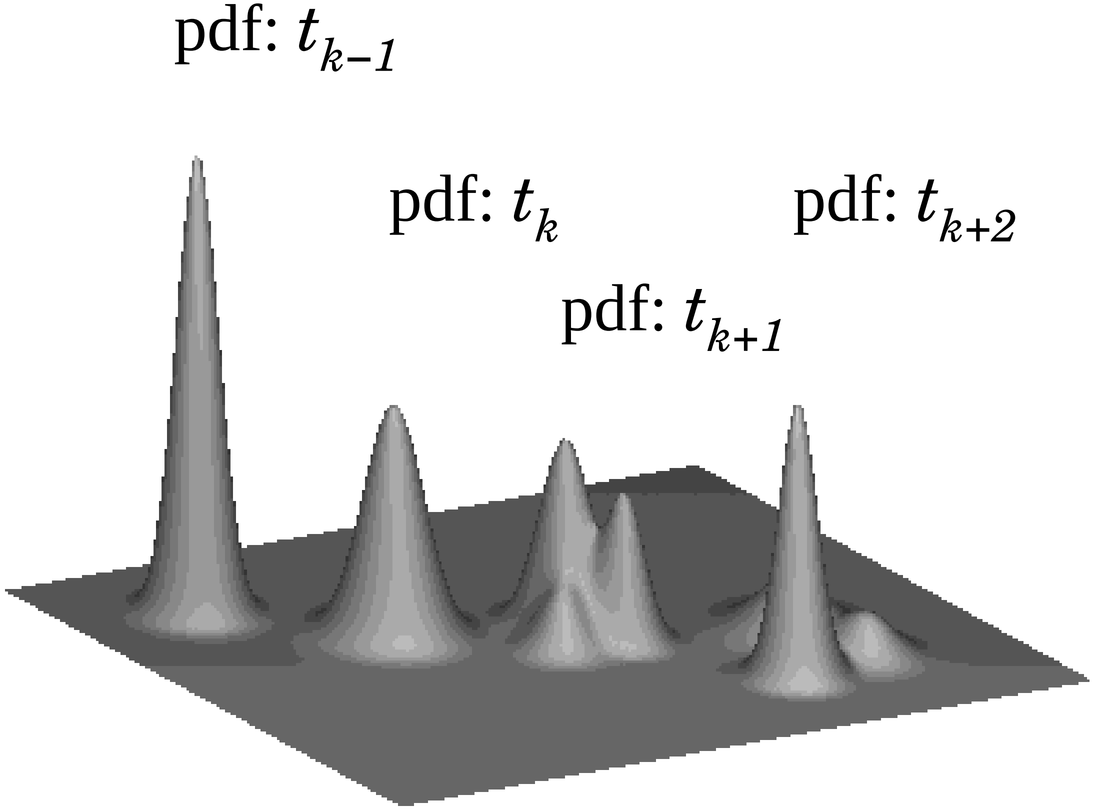


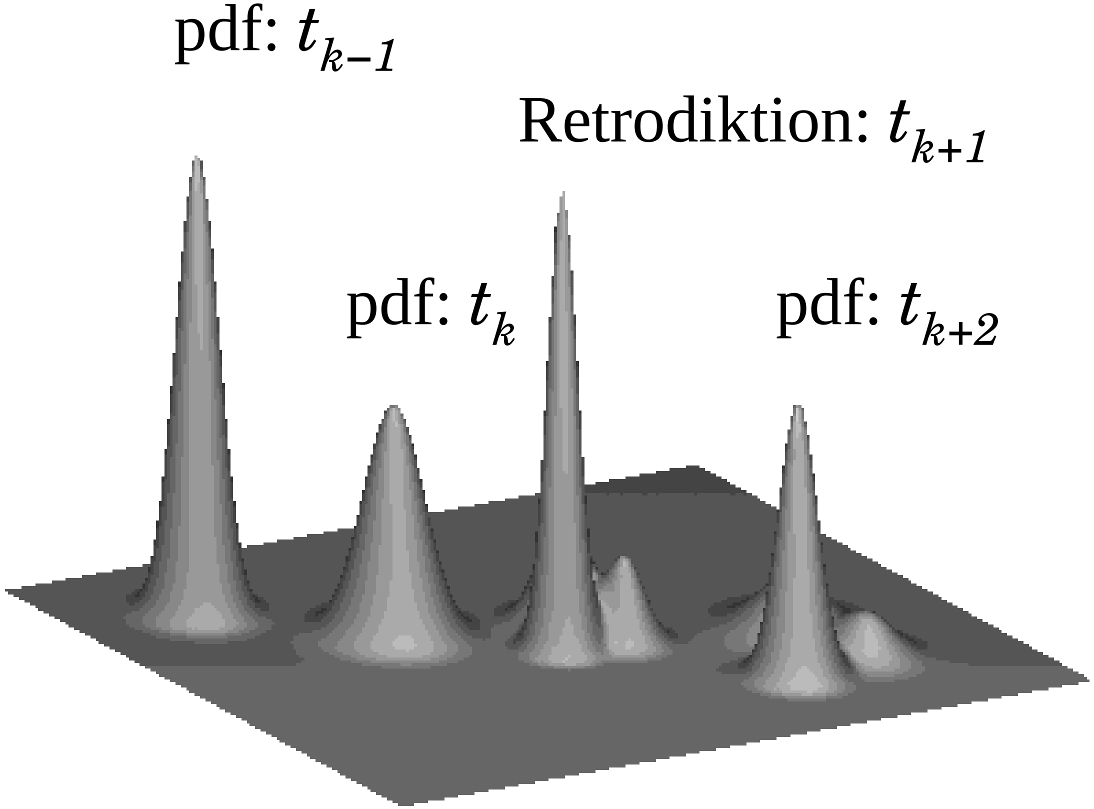
BAYES: $p(\mathbf{x}_{k+2}|\mathcal{Z}^{k+2}) = \frac{p(\mathbf{z}_{k+2}|\mathbf{x}_{k+2})\, p(\mathbf{x}_{k+2}|\mathcal{Z}^{k+1})}{\int d\mathbf{x}_{k+2}\, p(\mathbf{z}_{k+2}|\mathbf{x}_{k+2})\, p(\mathbf{x}_{k+2}|\mathcal{Z}^{k+1})}$



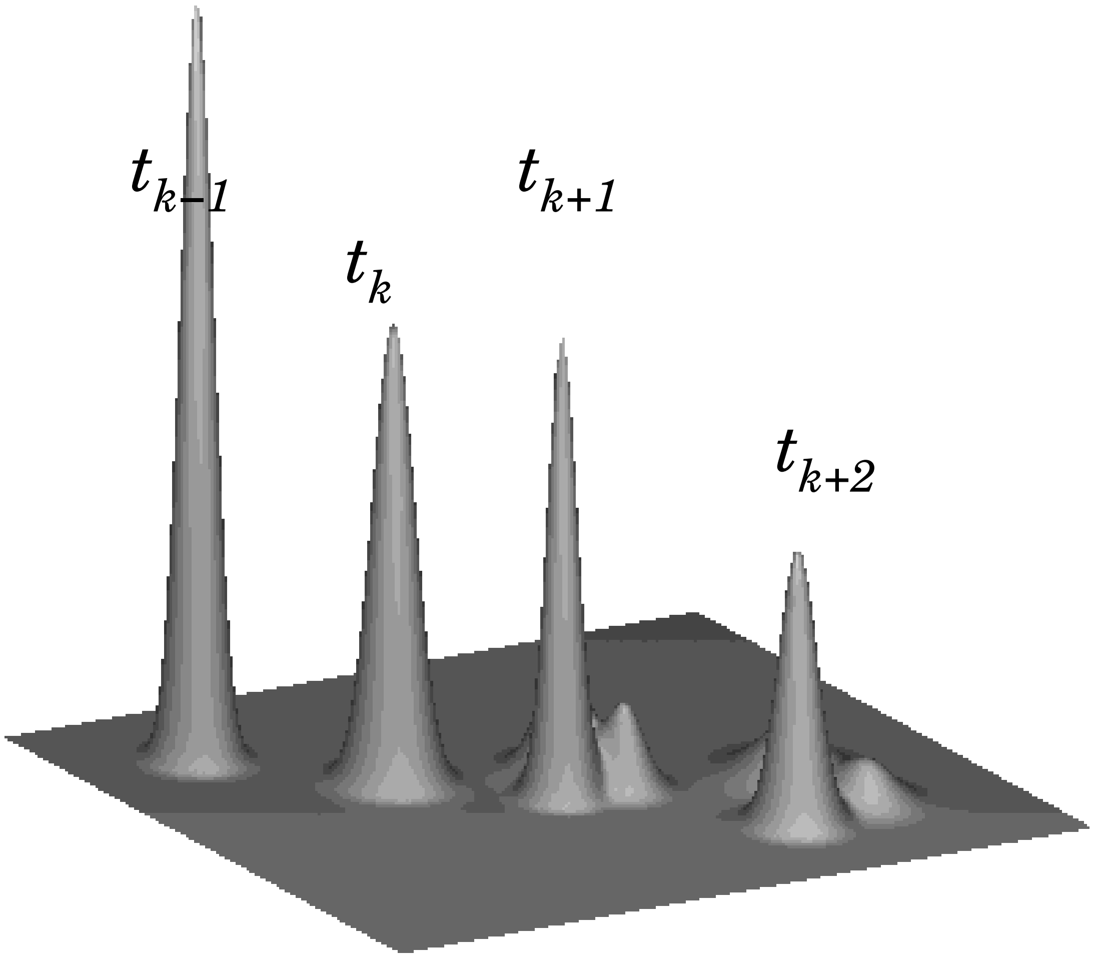
in particular: re-calculation of the hypothesis weights



How does new knowledge affect the knowledge in the past of a past state?



‘retrodiction’: a retrospective analysis of the past



optimal information processing at present and for the past

Multiple Hypothesis Tracking: Basic Idea

Iterative updating of conditional probability densities!

kinematic target state x_k at time t_k, **accumulated sensor data** $\mathcal{Z}^k$

a priori knowledge: target dynamics models, sensor model, road maps

• **prediction:**	$p(\mathbf{x}_{k-1}\|\mathcal{Z}^{k-1})$	$\xrightarrow[\text{road maps}]{\text{dynamics model}}$	$p(\mathbf{x}_k\|\mathcal{Z}^{k-1})$
• **filtering:**	$p(\mathrm{x}_k\|\mathcal{Z}^{k-1})$	$\xrightarrow[\text{sensor model}]{\text{sensor data } Z_k}$	$p(\mathrm{x}_k\|\mathcal{Z}^k)$
• **retrodiction:**	$p(\mathbf{x}_{l-1}\|\mathcal{Z}^k)$	$\xleftarrow[\text{dynamics model}]{\text{filtering output}}$	$p(\mathbf{x}_l\|\mathcal{Z}^k)$

– ***finite mixture:*** **inherent ambiguity (data, model, road *network*)**
– ***optimal estimators:*** **e.g. minimum mean squared error (MMSE)**
– ***initiation of pdf iteration:*** **multiple hypothesis track extraction**

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Difficult Operational Conditions

object detection:

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- fading: consecutive missing plots (interference)
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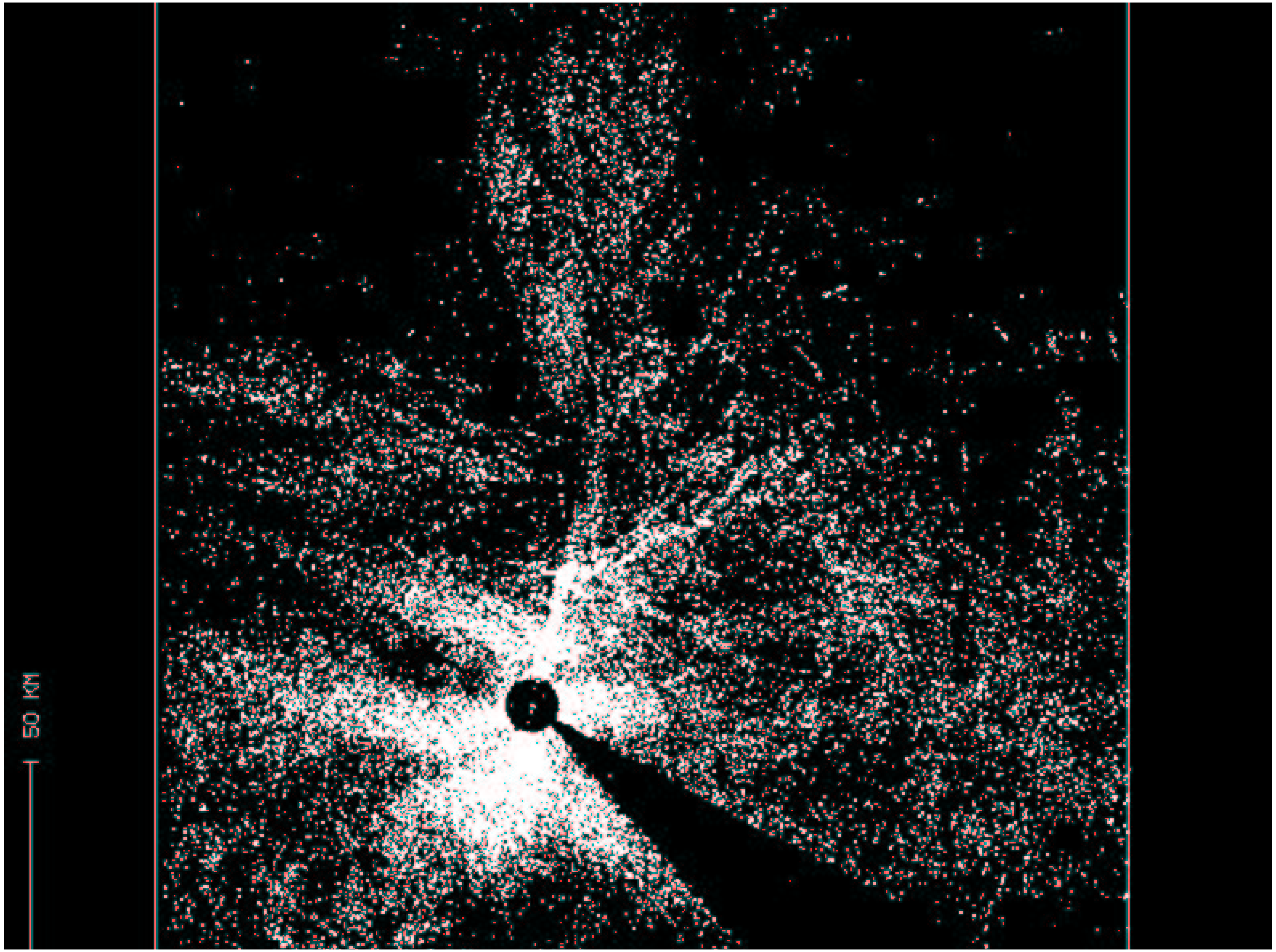
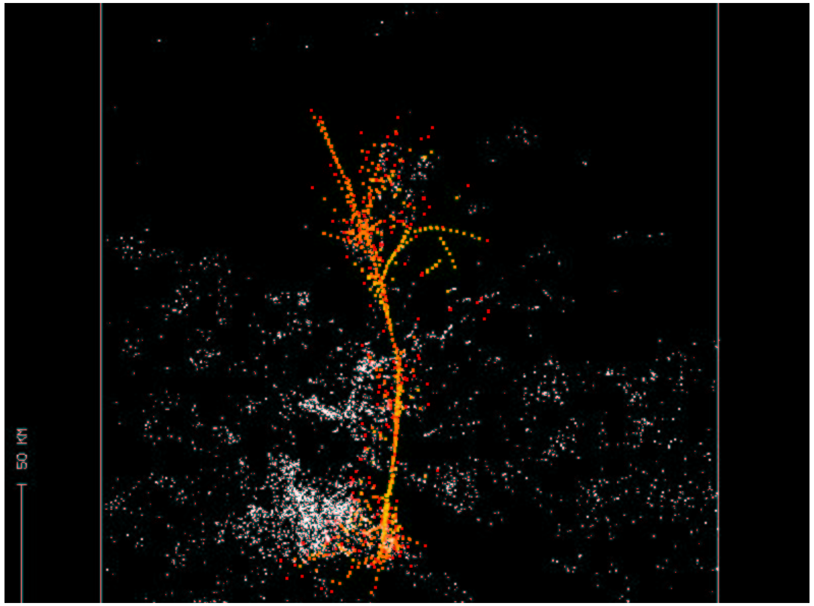
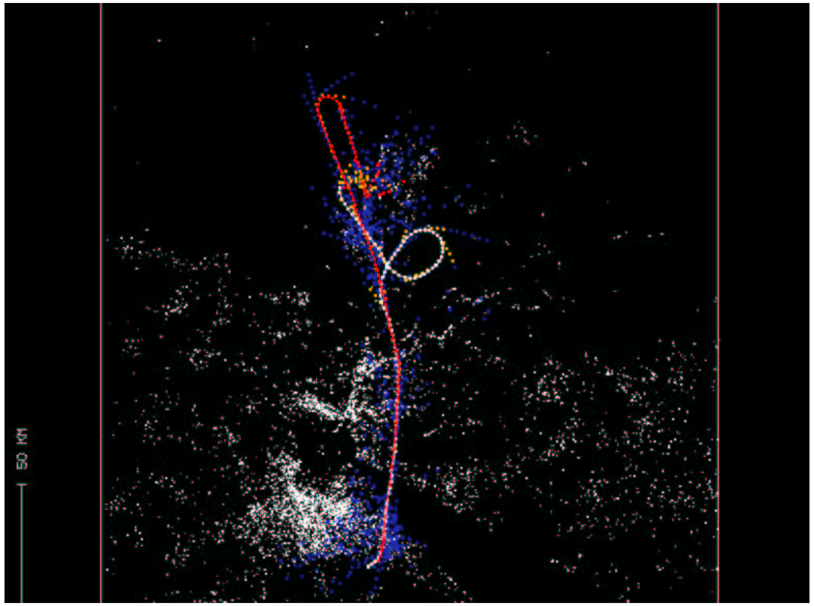
object behavior:

- applications: high maneuvering capability
- qualitatively distinct maneuvering phases
- dynamic object parameters a priori unknown

Demonstration: Multiple Hypothesis Tracking




50 KM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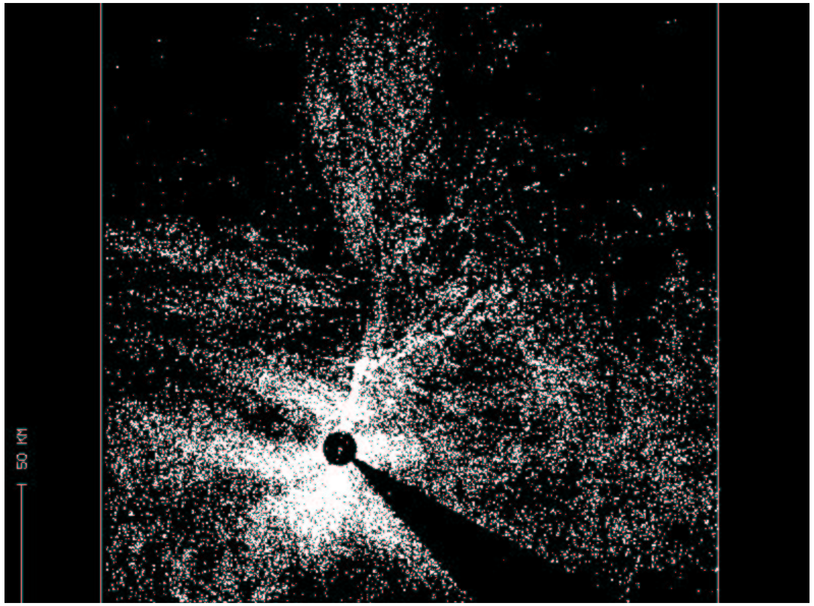
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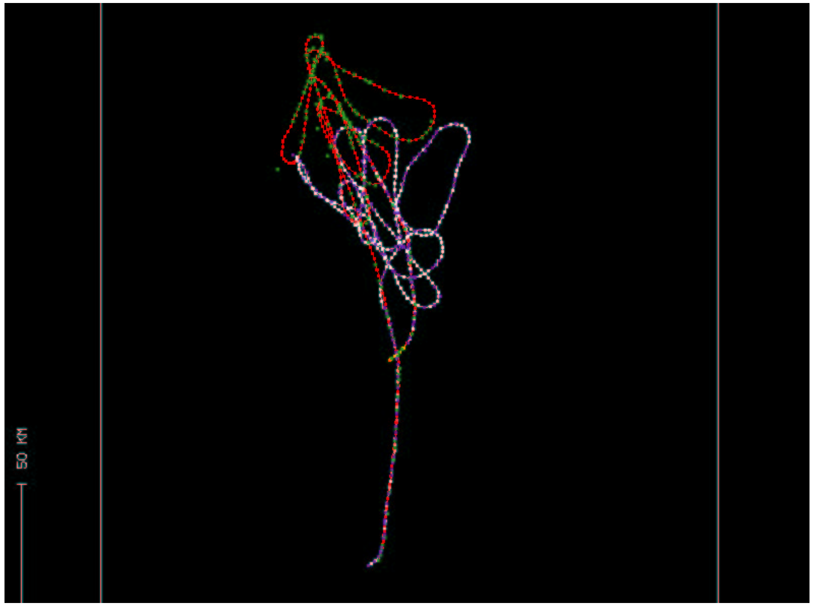
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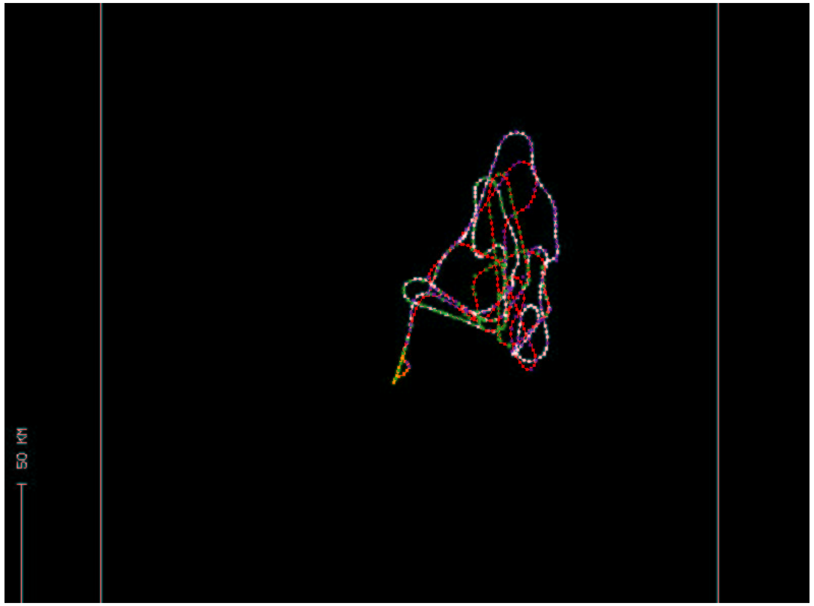
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- **Solution:** Derive iteration formulae for calculating the pdfs! Develop a mechanism for initiation! By doing so, exploit all background information available! Derive state estimates from the pdfs along with appropriate quality measures!

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How to deal with probability density functions?

- pdf $p(x)$: Extract *probability statements* about the RV x by integration!
- naïvely: *positive* and *normalized* functions ($p(x) \geq 0$, $\int dx\, p(x) = 1$)
- *conditional pdf* $p(x|y) = \frac{p(x,y)}{p(y)}$: Impact of information on y on RV x?
- *marginal density* $p(x) = \int dy\, p(x,y) = \int dy\, p(x|y)\, p(y)$: Enter y!
- Bayes: $p(x|y) = \frac{p(y|x)p(x)}{p(y)} = \frac{p(y|x)p(x)}{\int dx\, p(y|x)p(x)}$: $p(x|y) \leftarrow p(y|x), p(x)$!

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  $\int dx\, \delta(y - t[x])\, p_x(x) =: [\mathcal{T} p_x](y)$ ($\mathcal{T} : p_x \mapsto p$, "transfer operator")

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Create your own ground truth generator!

Exercise 2.1

Consider an object that moves in two dimensions on the trajectory:

$$\mathbf{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = A \begin{pmatrix} \sin(\omega t) \\ \sin(2\omega t) \end{pmatrix} \quad \text{with} \quad A = \frac{v^2}{q}, \quad \omega = \frac{q}{2v}$$

and speed and acceleration parameters: $v = 300\,\frac{\text{m}}{\text{s}}$, $q = 9\,\frac{\text{m}}{\text{s}^2}$!

1. Plot the trajectory. Why is it periodical? What is its period $T = T(v, q)$?
2. Show for the velocity and acceleration vector:

$$\dot{\mathbf{r}}(t) = v \begin{pmatrix} \cos(\omega t)/2 \\ \cos(2\omega t) \end{pmatrix}, \quad \ddot{\mathbf{r}}(t) = -q \begin{pmatrix} \sin(\omega t)/4 \\ \sin(2\omega t) \end{pmatrix}!$$

3. Calculate for each instance of time t the tangential and normal vectors in $\mathbf{r}(t)$:

$$\mathbf{t}(t) = \frac{1}{|\dot{\mathbf{r}}(t)|} \begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \end{pmatrix}, \quad \mathbf{n}(t) = \frac{1}{|\dot{\mathbf{r}}(t)|} \begin{pmatrix} -\dot{y}(t) \\ \dot{x}(t) \end{pmatrix}!$$

4. Plot $|\dot{\mathbf{r}}(t)|$, $|\ddot{\mathbf{r}}(t)|$, $\ddot{\mathbf{r}}(t)\mathbf{t}(t)$ and $\ddot{\mathbf{r}}(t)\mathbf{n}(t)$ over a period T!
5. Discuss the temporal behaviour based on the trajectory $\mathbf{r}(t)$!
6. What are the maximum speeds and accelerations, v_{max}, q_{max}?