Recapitulation: *Expected* Measurements

innovation statistics, expectation gates, gating

$$p(\mathbf{z}_k|\mathcal{Z}^{k-1}) = \int d\mathbf{x}_k\, p(\mathbf{z}_k, \mathbf{x}_k|\mathcal{Z}^{k-1}) = \int d\mathbf{x}_k\, p(\mathbf{z}_k|\mathbf{x}_k)\, p(\mathbf{x}_k|\mathcal{Z}^{k-1})$$

$$= \int d\mathbf{x}_k\, \underbrace{\mathcal{N}(\mathbf{z}_k;\, \mathbf{H}_k\mathbf{x}_k,\, \mathbf{R}_k)}_{\text{likelihood: sensor model}}\, \underbrace{\mathcal{N}(\mathbf{x}_k;\, \mathbf{x}_{k|k-1},\, \mathbf{P}_{k|k-1})}_{\text{prediction at time } t_k}$$

$$= \mathcal{N}(\mathbf{z}_k;\, \mathbf{H}_k\mathbf{x}_{k|k-1},\, \mathbf{S}_{k|k-1}) \qquad \text{(product formula)}$$

innovation: $\boldsymbol{\nu}_{k|k-1} = \mathbf{z}_k - \mathbf{H}_k\mathbf{x}_{k|k-1},$

innovation covariance: $\mathbf{S}_{k|k-1} = \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^\top + \mathbf{R}_k$

expectation gate: $\boldsymbol{\nu}_{k|k-1}^\top \mathbf{S}_{k|k-1}^{-1} \boldsymbol{\nu}_{k|k-1} \leq \lambda(P_c)$

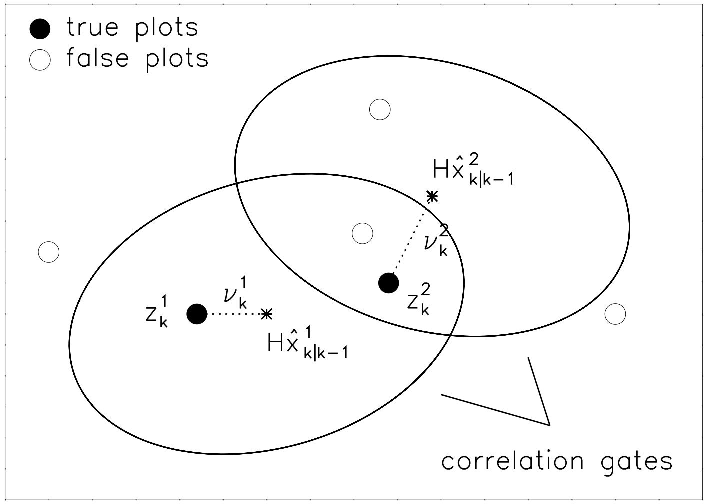
MAHALANOBIS — ellipsoid containing $\mathbf{z}_k$ with certain probability P_c

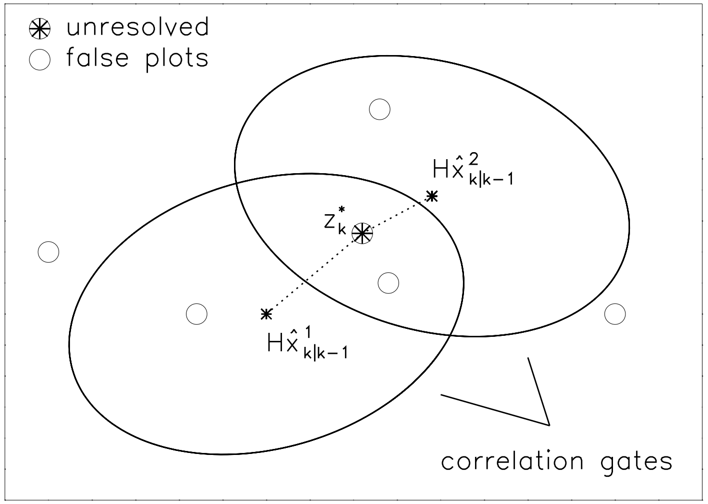
Choose $\lambda(P_c)$ (“gating parameter”) properly!

Can be looked up in a χ^2-table!

Fraunhofer FKIE

Sensor data of uncertain origin

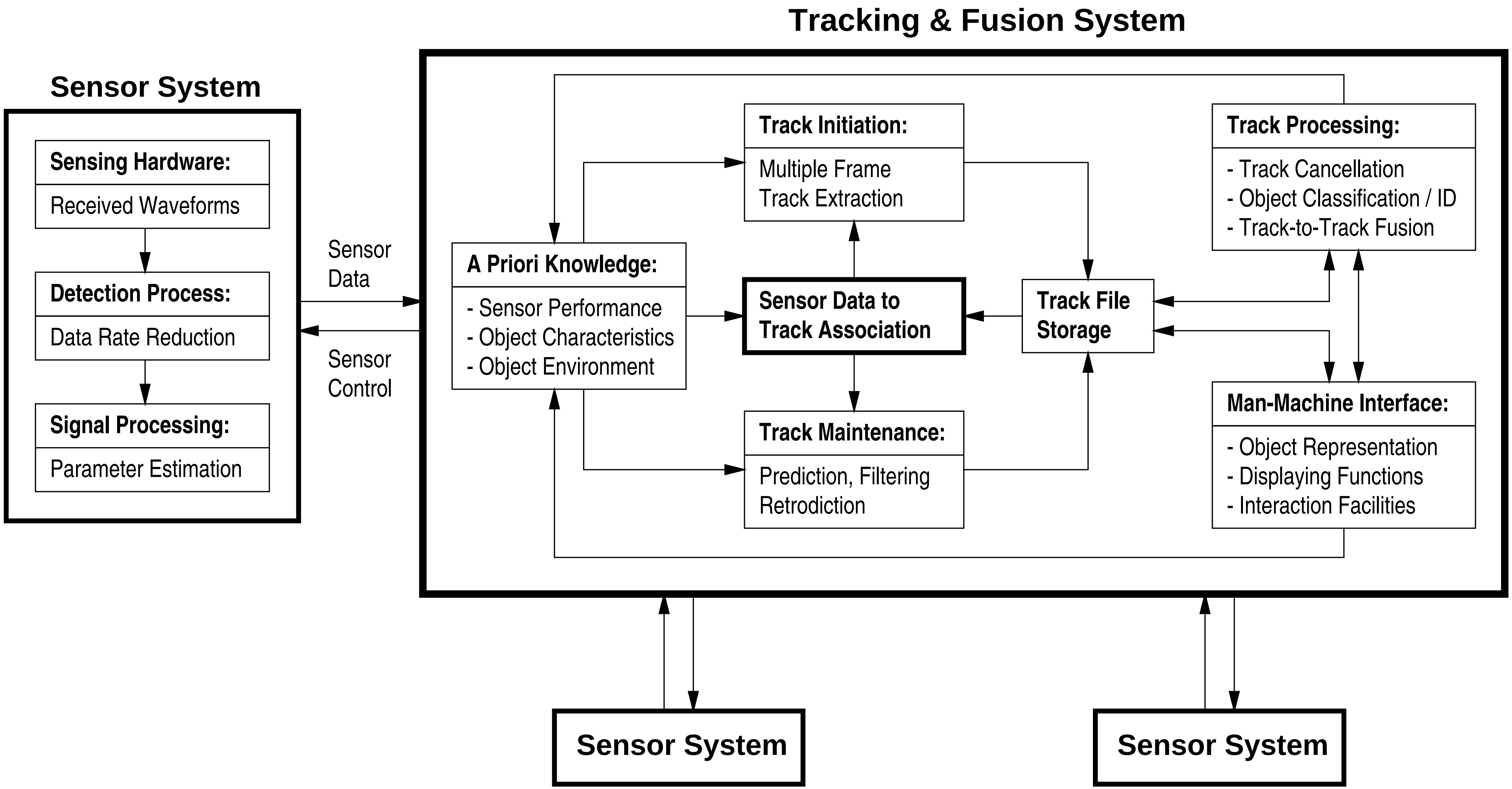




- prediction: $\mathbf{x}_{k|k-1}$, $\mathbf{P}_{k|k-1}$ (dynamics)
- innovation: $\boldsymbol{\nu}_k = \mathbf{z}_k - \mathbf{H}\mathbf{x}_{k|k-1}$, white
- Mahalanobis norm: $||\boldsymbol{\nu}_k||^2 = \boldsymbol{\nu}_k^\top \mathbf{S}_k^{-1} \boldsymbol{\nu}_k$
- expected plot: $\mathbf{z}_k \sim N(\mathbf{H}\mathbf{x}_{k|k-1}, \mathbf{S}_k)$
- $\boldsymbol{\nu}_k \sim N(0, \mathbf{S}_k)$, $\mathbf{S}_k = \mathbf{H}\mathbf{P}_{k|k-1}\mathbf{H}^\top + \mathbf{R}$
- gating: $||\boldsymbol{\nu}_k|| < \lambda$, $P_c(\lambda)$ correlation prob.

missing/false plots, measurement errors, scan rate, agile targets: large gates

A Generic Tracking and Sensor Data Fusion System



Fraunhofer FKIE

Description of the Detection Process

Detector: receives signals and decides on object existence

Processor: processes detected signals and produces measurements

‘D’: detector detects an object

D: object actually existent

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error of 1. kind: $P_{\mathrm{I}} = P(\neg\text{‘}D\text{’}|D)$

error of 2. kind: $P_{\mathrm{II}} = P(\text{‘}D\text{’}|\neg D)$

measure of detection performance: $P_D = P(\text{‘}D\text{’}|D)$

detector properties characterized by two parameters:

– detection probability $P_D = 1 - P_{\mathrm{I}}$
– false alarm probability $P_F = P_{\mathrm{II}}$

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example (Swerling I model): $P_D = P_D(P_F, \text{SNR}) = P_F^{1/(1+\text{SNR})}$

detector design: Maximize detection probability P_D for a given, predefined false alarm probability P_F!

ambiguous sensor data ($P_D < 1$, $\rho_F > 0$)

$n_k + 1$ possible interpretations of the sensor data $Z_k = \{\mathbf{z}_k^j\}_{j=1}^{n_k}$!

- E_0: the object was not detected; n_k false data in the Field of View (FoV)
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Consider the interpretations in the likelihood function $p(Z_k, n_k|\mathbf{x}_k)$!

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$$= p(Z_k, n_k|\neg D, \mathbf{x}_k) \underbrace{P(\neg D|\mathbf{x}_k)}_{=1-P_D(\mathbf{x}_k)} + p(Z_k, n_k|D, \mathbf{x}_k) \underbrace{P(D|\mathbf{x}_k)}_{=P_D(\mathbf{x}_k)}$$

sensor parameter: detection probability $P_D(\mathbf{x}_k)$

Fraunhofer FKIE

Tracking Application: Ground Picture Production

GMTI Radar: Ground Moving Target Indicator

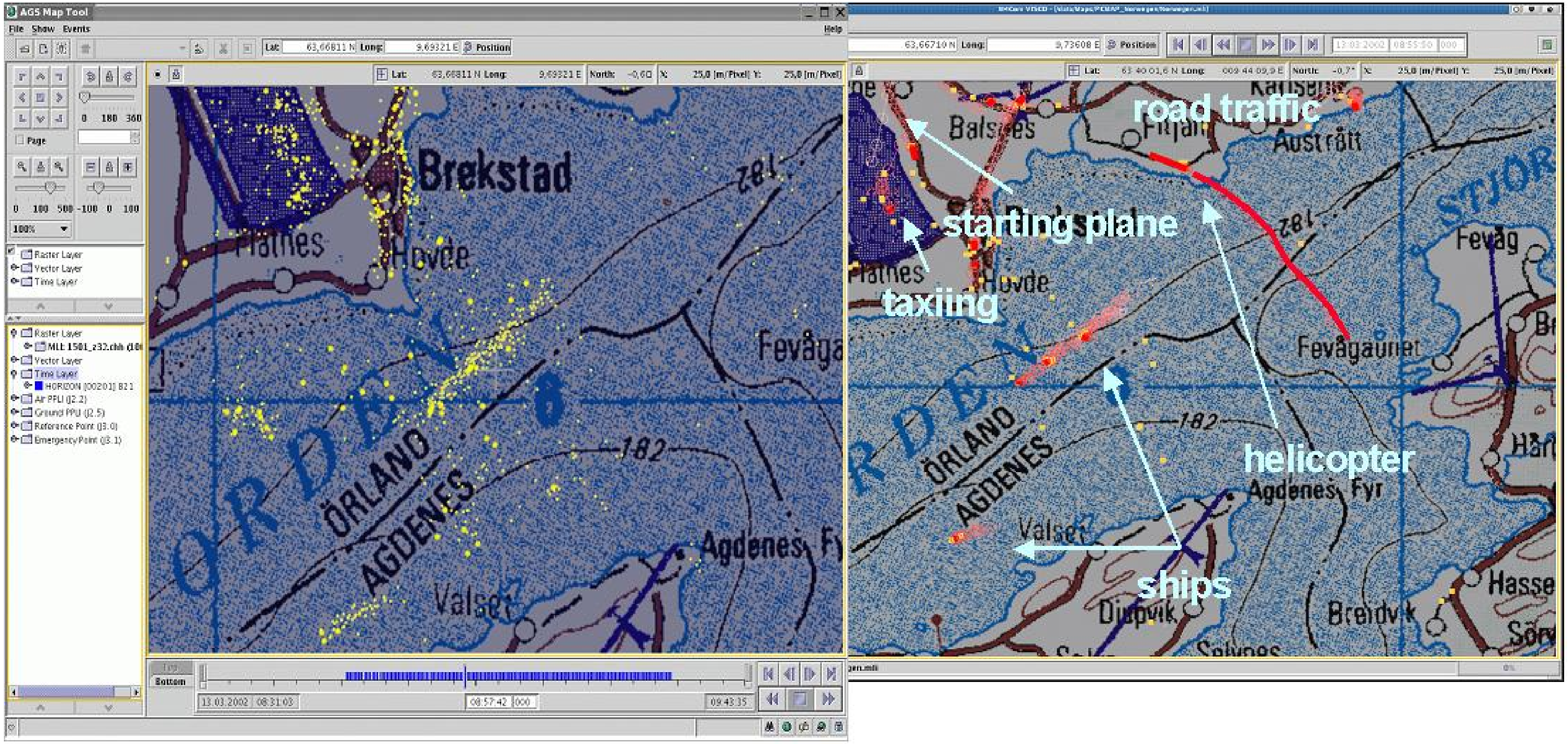
wide area, all-weather, day/night, real-time surveillance of a dynamically evolving ground or near-to-ground situation

GMTI Tracking: Some Characteristic Aspects

backbone of a ground picture: moving target tracks

- airborne, dislocated, mobile sensor platforms
- vehicles, ships, ‘low-flyers’, radars, convoys
- occlusions: Doppler-blindness, topography
- road maps, terrain information, tactical rules
- dense target / dense clutter situations: MHT

Examples of GMTI Tracks (live exercise)



Fraunhofer
FKIE

GMTI: Fusion of ‘Negative’ Information

low-DOPPLER targets can be masked by the GMTI clutter notch

- *fading:* series of missing plots (target/sensor geometry)
- *stopping targets:* indistinguishable from ground clutter
- *mdv:* minimum detectable velocity (sensor parameter)

a simple GMTI detection model: qualitative discussion

- detection depends on target kinematics & target/sensor geometry
- detection probability $P_D(\mathbf{x}_k)$ is small if $n_c = \dot{r}_k - \dot{r}_{\mathrm{C}}(\mathbf{x}_k) <$ *mdv*
- there exists a narrow transition region between these domains

sensor performance: *quantitative* model

- **basis:** $\mathsf{snir} = \mathsf{snir}(r_k, \varphi_k, \dot{r}_k)$, **Signal-to-Noise+Interference Ratio**

$$\mathsf{snir} = \mathsf{snir}_0 \underbrace{\left(\frac{\bar{\sigma}_k}{\sigma_0}\right)}_{\text{rcs}} \underbrace{\left(\frac{r_k}{r_0}\right)^{-4}}_{\text{propagation}} \underbrace{D(\varphi_k)}_{\text{directivity}} \underbrace{\left[1 - \mathsf{e}^{-\log 2 \left(\frac{n_c(r_k, \varphi_k, \dot{r}_k)}{v_m}\right)^2}\right]}_{\text{clutter notch } (<\frac{1}{2} \text{ fďż˝r } |n_c| < v_m)}$$

- **quadrature detector with given P_{FA}, rcs fluctuations: SWERLING I**

$$P_d(r_k, \varphi_k, \dot{r}_k) = P_{\mathsf{FA}}^{\frac{1}{1 + \mathsf{snir}(r_k, \varphi_k, \dot{r}_k)}}$$

- **as usual: residual clutter; bias free, GAUSSian errors (monopulse)**

$$\sigma_{r,\varphi,\dot{r}}(r_k, \varphi_k, \dot{r}_k) = \Sigma_{r,\varphi,\dot{r}} / \sqrt{\mathsf{snir}(r_k, \varphi_k, \dot{r}_k)}$$

Clutter Notch: A Priori Information!

– *current* position (Sensor-to-target-geometry)

– sensor specific *width* (STAP $\rightarrow$ MDV)

– detection process: *generic* model

GMTI model $\Rightarrow$ mixture densities:

– well-understood formalism directly applicable

– class of GAUSS*ian mixtures* remains invariant

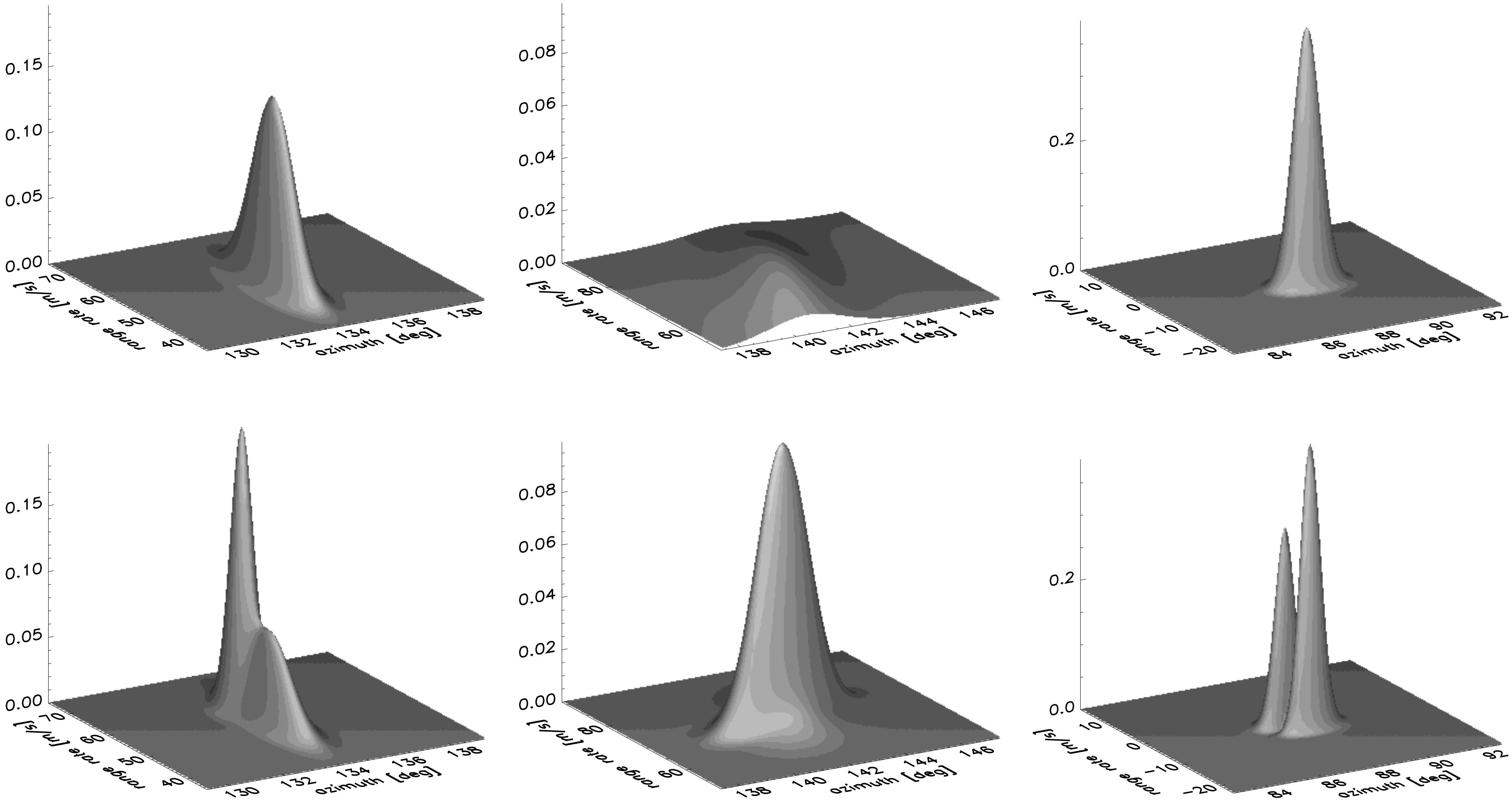
– pdfs characterized by a *set of parameters*

– growing memory: standard-type approximations

model inherent: **reason for missing detections**

$\Rightarrow$ An adequate treatment becomes possible!

PDFs with / without exploiting the GMTI sensor model

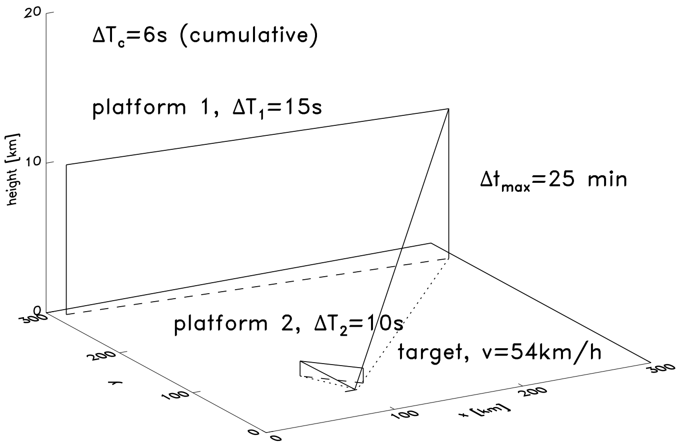


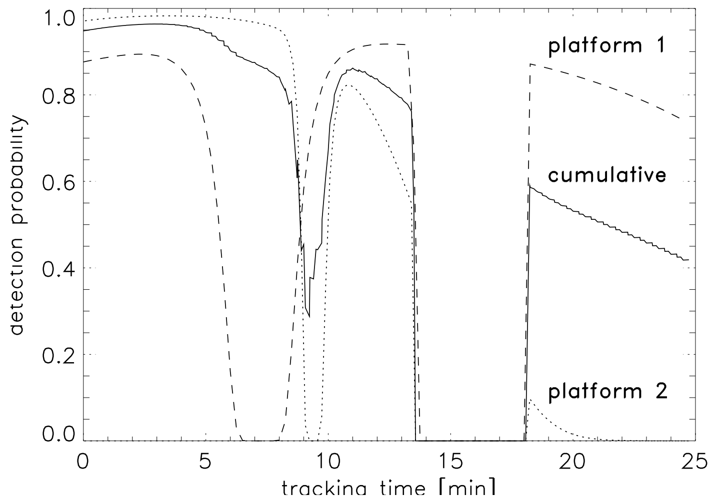
Missing detection occurred near the clutter notch

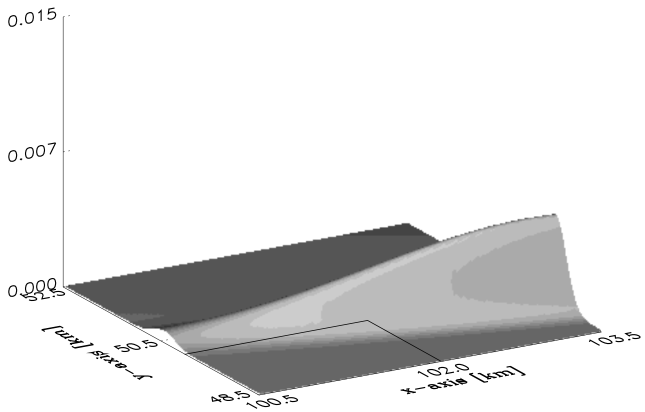
Several missing detections in the clutter notch

Detection occurred near the clutter notch

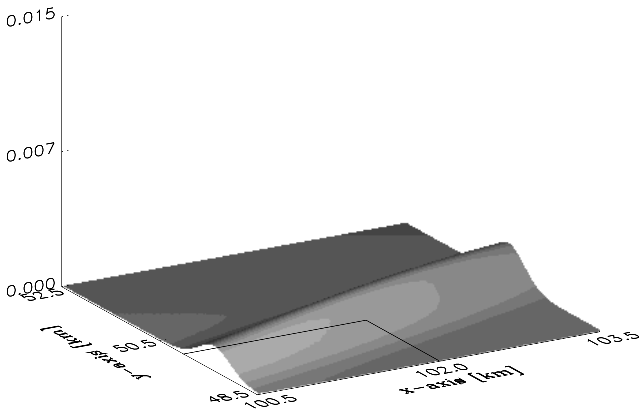
early detection of stopping targets



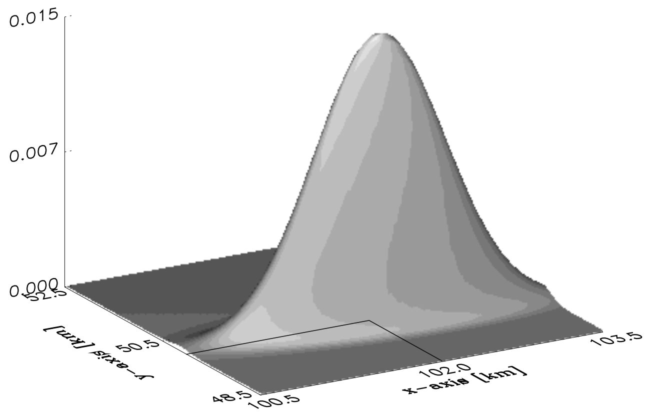




neg. output sensor 1



neg. output sensor 2



fusion: sensor 1+2

A *'negative' sensor output* can also provide information on the kinematical state vector of a target.

- ***fictitious plot*: function of position / radial speed**
- ***mdv*: appears as a fictitious measurement error**
- ***fusion*: exploit differing target/sensor geometries**

ambiguous sensor data ($P_D < 1$, $\rho_F > 0$)

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$$= p(Z_k, n_k|\neg D, \mathbf{x}_k)\, P(\neg D|\mathbf{x}_k) + p(Z_k, n_k|D, \mathbf{x}_k)\, p(D|\mathbf{x}_k)$$

$$= \underbrace{p(Z_k|n_k, \neg D, \mathbf{x}_k)}_{=|\mathrm{FoV}|^{-n_k}}\; \underbrace{p(n_k|\neg D, \mathbf{x}_k)}_{=p_F(n_k)}\, (1 - P_D) + P_D \sum_{j=1}^{n_k} p(Z_k, n_k, j|D, \mathbf{x}_k)$$

false measurements: Poisson distributed in #, uniformly distributed in the FoV

Modeling of False Measurements (FM)

- **Probability of having n FM:** $p_F(n)$
  - mean number of FM in the 'Field of View' (FoV) of a sensor:

    $\bar{n} = \rho_F|\mathsf{FoV}|$, false measurement density ρ_F (perhaps not constant)

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    expectation: $\mathbb{E}[n] = \bar{n}$, variance: $\mathbb{V}[n] = \bar{n}$

normalization: $\sum_{n=0}^{\infty} p_F(n) = \mathrm{e}^{-\bar{n}} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{n!} = \mathrm{e}^{-\bar{n}}\,\mathrm{e}^{\bar{n}}$ = 1

expectation: $\mathbb{E}[n] = \mathrm{e}^{-\bar{n}} \sum_{n=0}^{\infty} n \frac{\bar{n}^n}{n!} = \mathrm{e}^{-\bar{n}} \sum_{n=1}^{\infty} n \frac{\bar{n}^n}{n!} = \bar{n}\,\mathrm{e}^{-\bar{n}} \sum_{n=1}^{\infty} \frac{\bar{n}^{n-1}}{(n-1)!} =$
$\bar{n}$

variance: $\mathbb{V}[n] = \mathbb{E}[(n-\bar{n})^2] = \mathbb{E}[n^2] - \bar{n}^2 = \ldots \text{exercise!} \ldots = \bar{n}$

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- **Distribution of FM in the Field of View:** $p(\mathbf{z}_1^f, \ldots, \mathbf{z}_n^f|\textbf{FoV})$
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Fraunhofer FKIE

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  - uniformly distributed in the FoV: $p(\mathbf{z}_i^f|\mathsf{FoV}) = |\mathsf{FoV}|^{-1}$ (often!)

ambiguous sensor data ($P_D < 1$, $\rho_F > 0$)

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$$= |\mathsf{FoV}|^{-n_k}\, p_F(n_k)\, (1 - P_D) + P_D \sum_{j=1}^{n_k} \underbrace{p(Z_k|n_k, j, D, \mathbf{x}_k)}_{|\mathsf{FoV}|^{-(n_k-1)} \mathcal{N}(\mathbf{z}_k^j; \mathbf{H}\mathbf{x}_k, \mathbf{R})} \underbrace{p(j|n_k, D)}_{=1/n_k} \underbrace{p(n_k|D)}_{=p_F(n_k-1)}$$

Insert Poisson distribution: $\quad p_F(n_k) = \frac{(\rho_F|\mathsf{FoV}|)^{-n_k}}{n_k!}\, \mathrm{e}^{-\rho_F|\mathsf{FoV}|}$

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$$= |\mathrm{FoV}|^{-n_k}\, p_F(n_k)\, (1 - P_D) + P_D \sum_{j=1}^{n_k} p(Z_k|n_k, j, D, \mathbf{x}_k)\, p(j|n_k, D)\, p(n_k|D)$$

$$= \frac{\mathrm{e}^{-\rho_F|\mathrm{FoV}|}}{n_k!}\, \rho_F^{n_k-1} \Big((1 - P_D)\rho_F + P_D \sum_{j=1}^{n_k} \mathcal{N}\big(\mathbf{z}_k^j;\, \mathbf{H}\mathbf{x}_k,\, \mathbf{R}\big)\Big)$$

Likelihood Functions

The likelihood function answers the question:

What does the sensor tell about the state $\mathbf{x}$ of the object?

(input: sensor data, sensor model)

- **ideal conditions, one object:** $P_D = 1$**,** $\rho_F = 0$

  at each time one measurement: $$p(\mathbf{z}_k|\mathbf{x}_k) = \mathcal{N}(\mathbf{z}_k;\mathbf{H}\mathbf{x}_k,\mathbf{R})$$

- **real conditions, one object:** $P_D < 1$**,** $\rho_F > 0$

  at each time n_k measurements $Z_k = \{\mathbf{z}_k^1, \ldots, \mathbf{z}_k^{n_k}\}$!

$$p(Z_k, n_k|\mathbf{x}_k) \propto (1 - P_D)\rho_F + P_D \sum_{j=1}^{n_k} \mathcal{N}\big(\mathbf{z}_k^j;\, \mathbf{H}\mathbf{x}_k,\, \mathbf{R}\big)$$

PDAF Filter: formally analogous to Kalman Filter

Filtering (scan $k-1$): $p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1}) \approx \mathcal{N}(\mathbf{x}_{k-1};\, \mathbf{x}_{k-1|k-1}, \mathbf{P}_{k-1|k-1})$ ($\rightarrow$ initiation)

prediction (scan k): $p(\mathbf{x}_{k}|\mathcal{Z}^{k-1}) \approx \mathcal{N}(\mathbf{x}_{k};\, \mathbf{x}_{k|k-1}, \mathbf{P}_{k|k-1})$ (like Kalman)

Filtering (scan k): $p(\mathbf{x}_{k}|\mathcal{Z}^{k}) \approx \sum_{j=0}^{m_k} p_k^j\, \mathcal{N}(\mathbf{x}_{k};\, \mathbf{x}_{k|k}^j, \mathbf{P}_{k|k}^j)$

BLACKBOARD!

Fraunhofer FKIE

PDAF Filter: formally analogous to Kalman Filter

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$$\mathbf{x}_{k|k}^j = \begin{cases} \mathbf{x}_{k|k-1} & j=0 \\ \mathbf{x}_{k|k-1} + \mathbf{W}_k \boldsymbol{\nu}_k^j & j\neq 0 \end{cases} \qquad \mathbf{P}_{k|k}^j = \begin{cases} \mathbf{P}_{k|k-1} & j=0 \\ \mathbf{P}_{k|k-1} - \mathbf{W}_k \mathbf{S}_k \mathbf{W}_k^\top & j\neq 0 \end{cases}$$

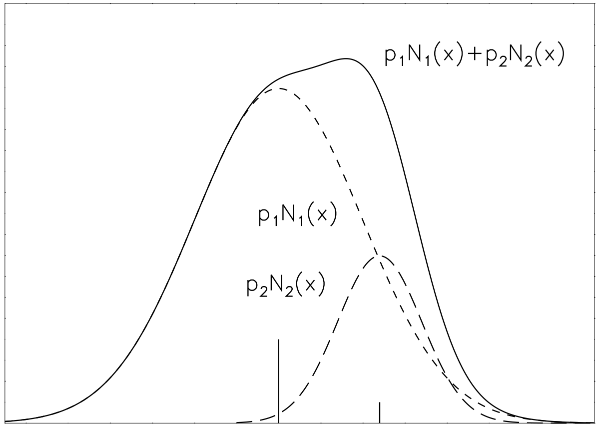
$$\boldsymbol{\nu}_k^j = \underbrace{\mathbf{z}_k^j - \mathbf{H}\mathbf{x}_k}_{\text{innovation}}, \qquad \mathbf{W}_k = \underbrace{\mathbf{P}_{k|k-1}\mathbf{H}^\top \mathbf{S}_k^{-1}}_{\text{gain matrix}}, \qquad \mathbf{S}_k = \underbrace{\mathbf{H}\mathbf{P}_{k|k-1}\mathbf{H}^\top + \mathbf{R}_k}_{\text{innovation covariance}}$$

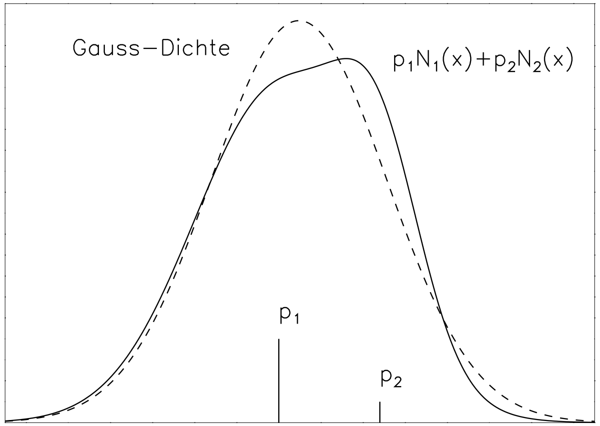
$$p_k^j = \underbrace{\frac{p_k^{j*}}{\sum_j p_k^{j*}}}_{\text{Gewichte}}, \qquad p_k^{j*} = \begin{cases} (1-P_D)\,\rho_F & j=0 \\ \frac{P_D}{\sqrt{|2\pi \mathbf{S}_k|}}\, \mathrm{e}^{-\frac{1}{2}\boldsymbol{\nu}_{H_k}^\top \mathbf{S}_k^{-1} \boldsymbol{\nu}_{H_k}} & j\neq 0 \end{cases}$$

Moment Matching: Approximate an arbitrary pdf

$p(x)$ with $\mathbb{E}[x] = \mathbf{x}$, $\mathbb{C}[x] = \mathbf{P}$ by $p(x) \approx \mathcal{N}(x;\, \mathbf{x},\, \mathbf{P})$!

here especially: $p(x) = \sum_H p_H \, \mathcal{N}(x; \mathbf{x}_H, \mathbf{P}_H)$ (normal mixtures)





$$\mathbf{x} = \sum_H p_H \, \mathbf{x}_H$$

$$\mathbf{P} = \sum_H p_H \left\{ \mathbf{P}_H + \overbrace{(\mathbf{x}_H - \mathbf{x})(\mathbf{x}_H - \mathbf{x})^\top}^{\text{spread term}} \right\}$$

Fraunhofer FKIE

Second-order Approximation of the Mixture Density:

$$\sum_{j=1}^{m_k} p_k^j \, \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{k|k}^j,\, \mathbf{P}_{k|k}^j\big) \approx \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{k|k},\, \mathbf{P}_{k|k}\big)$$

mit:

$$\mathbf{x}_{k|k} = \sum_{j=0}^{m_k} p_k^j \, \mathbf{x}_{k|k}^j$$

$$\mathbf{P}_{k|k} = \sum_{j=0}^{m_k} p_k^j \, \big(\mathbf{P}_{k|k}^j + (\mathbf{x}_{k|k}^j - \mathbf{x}_{k|k})(\mathbf{x}_{k|k}^j - \mathbf{x}_{k|k})^\top\big)$$

$$\mathbf{x}_{k|k} = \sum_{j=0}^{m_k} p_k^j \mathbf{x}_{k|k}^j, \qquad \mathbf{x}_{k|k}^0 = \mathbf{x}_{k|k-1}, \ \mathbf{x}_{k|k}^j = \mathbf{x}_{k|k-1} + \mathbf{W}_k \boldsymbol{\nu}_k^j$$

$$\mathbf{P}_{k|k} = \sum_{j=0}^{m_k} p_k^j \Big(\mathbf{P}_{k|k}^j + (\mathbf{x}_{k|k}^j - \mathbf{x}_{k|k})(\mathbf{x}_{k|k}^j - \mathbf{x}_{k|k})^\top \Big)$$

$$\mathbf{x}_{k|k} = \sum_{j=0}^{m_k} p_k^j \mathbf{x}_{k|k}^j$$

$$= p_k^0 \mathbf{x}_{k|k-1} + \sum_{j=1}^{m_k} p_k^j \big(\mathbf{x}_{k|k-1} + \mathbf{W}_k \boldsymbol{\nu}_k^j\big)$$

$$\mathbf{P}_{k|k} = \sum_{j=0}^{m_k} p_k^j \big(\mathbf{P}_{k|k}^j + (\mathbf{x}_{k|k}^j - \mathbf{x}_{k|k})(\mathbf{x}_{k|k}^j - \mathbf{x}_{k|k})^\top\big)$$

$$\mathbf{x}_{k|k} = \sum_{j=0}^{m_k} p_k^j \mathbf{x}_{k|k}^j$$

$$= p_k^0 \mathbf{x}_{k|k-1} + \sum_{j=1}^{m_k} p_k^j \big(\mathbf{x}_{k|k-1} + \mathbf{W}_k \boldsymbol{\nu}_k^j\big)$$

$$= \mathbf{x}_{k|k-1} \underbrace{\left(p_k^0 + \sum_{j=1}^{m_k} p_k^j\right)}_{=1!} + \mathbf{W}_k \underbrace{\sum_{j=1}^{m_k} p_k^j \boldsymbol{\nu}_k^j}_{\text{mean!}}$$

$$\mathbf{P}_{k|k} = \sum_{j=0}^{m_k} p_k^j \big(\mathbf{P}_{k|k}^j + (\mathbf{x}_{k|k}^j - \mathbf{x}_{k|k})(\mathbf{x}_{k|k}^j - \mathbf{x}_{k|k})^\top\big)$$

$$\mathbf{x}_{k|k} = \sum_{j=0}^{m_k} p_k^j \mathbf{x}_{k|k}^j$$

$$= p_k^0 \mathbf{x}_{k|k-1} + \sum_{j=1}^{m_k} p_k^j \big(\mathbf{x}_{k|k-1} + \mathbf{W}_k \boldsymbol{\nu}_k^j\big) = \mathbf{x}_{k|k-1} + \mathbf{W}_k \, \boldsymbol{\nu}_k$$

$$\mathbf{P}_{k|k} = \sum_{j=0}^{m_k} p_k^j \big(\mathbf{P}_{k|k}^j + (\mathbf{x}_{k|k}^j - \mathbf{x}_{k|k})(\mathbf{x}_{k|k}^j - \mathbf{x}_{k|k})^\top\big)$$

***Combined* Innovation:** $\boldsymbol{\nu}_k = \sum_{j=1}^{m_k} p_k^j \boldsymbol{\nu}_k^j$

$$\mathbf{x}_{k|k} = \sum_{j=0}^{m_k} p_k^j \mathbf{x}_{k|k}^j$$

$$= p_k^0 \mathbf{x}_{k|k-1} + \sum_{j=1}^{m_k} p_k^j \big(\mathbf{x}_{k|k-1} + \mathbf{W}_k \boldsymbol{\nu}_k^j\big) = \mathbf{x}_{k|k-1} + \mathbf{W}_k \, \boldsymbol{\nu}_k$$

$$\mathbf{P}_{k|k} = \sum_{j=0}^{m_k} p_k^j \big(\mathbf{P}_{k|k}^j + (\mathbf{x}_{k|k}^j - \mathbf{x}_{k|k})(\mathbf{x}_{k|k}^j - \mathbf{x}_{k|k})^\top\big), \qquad \mathbf{P}_{k|k}^0 = \mathbf{P}_{k|k-1}, \; \mathbf{P}_{k|k}^j = \mathbf{P}_{k|k-1} - \mathbf{W}_k \mathbf{S}_k \mathbf{W}_k^\top$$

$$= \mathbf{P}_{k|k-1} - \sum_{j=1}^{m_k} p_k^j \mathbf{W}_k \mathbf{S}_k \mathbf{W}_k^\top + \sum_{j=1}^{m_k} p_k^j \mathbf{W}_k (\boldsymbol{\nu}_k^j - \boldsymbol{\nu}_k)(\boldsymbol{\nu}_k^j - \boldsymbol{\nu}_k)^\top \mathbf{W}_k^\top$$

***Combined* Innovation:** $\boldsymbol{\nu}_k = \sum_{j=1}^{m_k} p_k^j \boldsymbol{\nu}_k^j$

$$\mathbf{x}_{k|k} = \sum_{j=0}^{m_k} p_k^j \mathbf{x}_{k|k}^j$$

$$= p_k^0 \mathbf{x}_{k|k-1} + \sum_{j=1}^{m_k} p_k^j \big(\mathbf{x}_{k|k-1} + \mathbf{W}_k \boldsymbol{\nu}_k^j\big) = \mathbf{x}_{k|k-1} + \mathbf{W}_k\, \boldsymbol{\nu}_k$$

$$\mathbf{P}_{k|k} = \sum_{j=0}^{m_k} p_k^j \big(\mathbf{P}_{k|k}^j + (\mathbf{x}_{k|k}^j - \mathbf{x}_{k|k})(\mathbf{x}_{k|k}^j - \mathbf{x}_{k|k})^\top\big)$$

$$= \mathbf{P}_{k|k-1} - \sum_{j=1}^{m_k} p_k^j \mathbf{W}_k \mathbf{S}_k \mathbf{W}_k^\top + \sum_{j=1}^{m_k} p_k^j \mathbf{W}_k (\boldsymbol{\nu}_k^j - \boldsymbol{\nu}_k)(\boldsymbol{\nu}_k^j - \boldsymbol{\nu}_k)^\top \mathbf{W}_k^\top$$

$$= \mathbf{P}_{k|k-1} - (1 - p_k^0) \mathbf{W}_k \mathbf{S}_k \mathbf{W}_k^\top + \mathbf{W}_k \Big[\sum_{j=1}^{m_k} p_k^j \boldsymbol{\nu}_k^j \boldsymbol{\nu}_k^{j\top} - \boldsymbol{\nu}_k \boldsymbol{\nu}_k{}^\top\Big] \mathbf{W}_k^\top$$

***Combined* Innovation:** $\boldsymbol{\nu}_k = \sum_{j=1}^{m_k} p_k^j \boldsymbol{\nu}_k^j$

PDAF Filter: formally analog to Kalman Filter

Filtering (scan $k-1$): $p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1}) \approx= \mathcal{N}(\mathbf{x}_{k-1};\, \mathbf{x}_{k-1|k-1}, \mathbf{P}_{k-1|k-1})$ (→ initiation)

prediction (scan k): $p(\mathbf{x}_k|\mathcal{Z}^{k-1}) \approx \mathcal{N}(\mathbf{x}_k;\, \mathbf{x}_{k|k-1}, \mathbf{P}_{k|k-1})$ (like Kalman)

Filtering (scan k): $p(\mathbf{x}_k|\mathcal{Z}^k) \approx \sum_{j=0}^{m_k} p_k^j \mathcal{N}(\mathbf{x}_k;\, \mathbf{x}_{k|k}^j, \mathbf{P}_{k|k}^j) \approx \mathcal{N}(\mathbf{x}_k;\, \mathbf{x}_{k|k}, \mathbf{P}_{k|k})$

$$\boldsymbol{\nu}_k = \boxed{\textstyle\sum_{j=0}^{m_k} p_k^j\, \boldsymbol{\nu}_k^j}, \quad \boldsymbol{\nu}_k^j = \mathbf{z}_k^j - \mathbf{H}\mathbf{x}_{k|k-1}$$

combined innovation

$$\mathbf{W}_k = \mathbf{P}_{k|k-1}\mathbf{H}^\top\mathbf{S}_k^{-1}, \quad \mathbf{S}_k = \mathbf{H}\mathbf{P}_{k|k-1}\mathbf{H}^\top + \mathbf{R}_k$$

Kalman gain matrix

$$p_k^j = p_k^{j*} \,/\, \textstyle\sum_j p_k^{j*}, \quad p_k^{j*} = \begin{cases} (1-P_D)\,\rho_F \\ \frac{P_D}{\sqrt{|2\pi\mathbf{S}_{H_k}|}}\, \mathrm{e}^{-\frac{1}{2}\boldsymbol{\nu}_{H_k}^\top \mathbf{S}_{H_k}\boldsymbol{\nu}_{H_k}} \end{cases}$$

weighting factors

$$\mathbf{x}_k = \mathbf{x}_{k|k-1} + \mathbf{W}_k\, \boldsymbol{\nu}_k$$

(Filtering Update: Kalman)

$$\mathbf{P}_k = \mathbf{P}_{k|k-1} - (1-p_k^0)\, \mathbf{W}_k \mathbf{S} \mathbf{W}_k^\top$$

(Kalman part)

$$+\, \mathbf{W}_k \Big\{ \textstyle\sum_{j=0}^{m_k} p_k^j\, \boldsymbol{\nu}_k^j \boldsymbol{\nu}_k^{j\top} - \boldsymbol{\nu}_k \boldsymbol{\nu}_k{}^\top \Big\} \mathbf{W}_k^\top$$

(Spread of Innovations)

PDAF: Characteristic Properties

- filtering: processing of *combined innovation*
- *all data* Z_k in the gate are considered
- p_i data dependent! Update *not linear*
- missing measurement: $\mathbf{P}_{k|k-1}$ with weight p_0
- *"usual"* Kalman covarianve according to $(1 - p_0)$
- Spread *positively semidefinite:* larger covariance
- therefore: *data driven adaptivity*
- *non linear estimator:* data dependent error
- Performance prediction *only via simulations*

Multimodality is lost! What about multiple sensor data?

Cumulative Detection by N Sensors

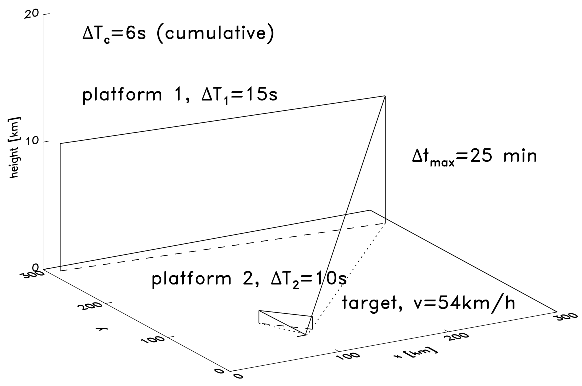
***cumulative* detection probability** $$P_D^{\text{kum}}(N) = 1 - \prod_{n=1}^{N}(1 - P_D^n)$$

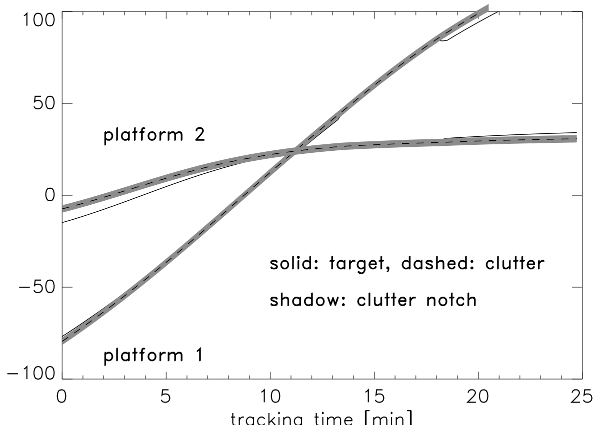
example: Doppler blindness in case of GMTI radar

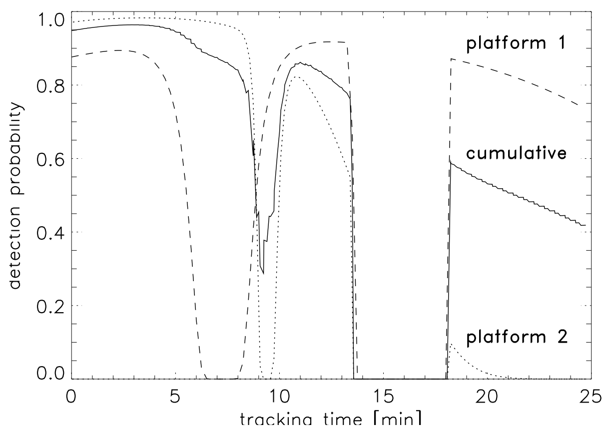
Cumulative Detection by N Sensors

cumulative **detection probability** $$P_D^{\text{kum}}(N) = 1 - \prod_{n=1}^{N} (1 - P_D^n)$$

example: Doppler blindness in case of GMTI radar







mean cumulative revisit interval: $$1/\Delta T_c = \sum_{n=1}^{N} 1/\Delta T_n$$

mean cumulative P_D **relative to** ΔT_c**:** $$P_D^c = 1 - \prod_{n=1}^{N} (1 - P_D^n)^{\Delta T_c/\Delta T_n}$$

Fraunhofer FKIE